Effective Field Theory For QCD-Like Theories in TeV scale

Jie Lu

IFIC, Valencia University, Spain

The 9th Workshop of TeV Physics Working Group in 2014
Sun Yat-sen (Zhongshan) University, Guangzhou, China
May 16, 2014

1. MOTIVATION

2. EFFECTIVE FIELD THEORY FOR QCD-LIKE THEORIES
   - Quark level Theories
   - Effective Theories

3. THE CALCULATIONS
   - 1. mass, decay constant and quark condensates
   - 2. General Meson-Meson Scattering
   - 3. Two Point Functions and S-parameter

Motivation

Our motivations for studying Effective Field Theory of QCD-like Theory:

- Systematically develop a theoretical framework for studying the low energy theory of the strong dynamic in TeV scale;
- Provide the high order analytic results for Chiral extrapolation in Lattice computing;

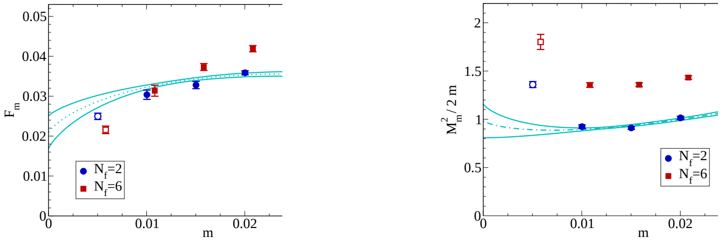


Figure: Plots from T. Appelquist *et al.*, arXiv:0910.2224.

- Help to understand the mechanism of diquark condensate in finite baryon density.

Quarks live in the complex representation.

- The n_f flavor QCD Lagrangian with external source terms

$$\begin{aligned} \mathcal{L} &= \bar{q}_L i\gamma^\mu D_\mu q_L + \bar{q}_R i\gamma^\mu D_\mu q_R \\ &\quad + \bar{q}_L \gamma^\mu l_\mu q_L + \bar{q}_R \gamma^\mu r_\mu q_R - \bar{q}_L \mathcal{M}^\dagger q_R - \bar{q}_R \mathcal{M} q_L \\ D_\mu &= \partial_\mu q - iG_\mu q, \qquad i,j = 1,2,...,n_f \\ \mathcal{M} &= \mathrm{diag}(\hat{m},...,\hat{m}) \end{aligned}$$

- If $\hat{m} = 0$, the flavor symmetry is $SU(n_f)_L \times SU(n_f)_R$.
- The vacuum condensate $\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$, which spontaneously breaks the $SU(n_f)_L \times SU(n_f)_R$ down to $SU(n_f)_V$.
- The small quark mass $\hat{m} \neq 0$ explicitly break the flavor symmetry $SU(n_f)_L \times SU(n_f)_R \to SU(n_f)_V$.

Quarks live in real representation

- The Lagrangian with external source terms

$$\begin{aligned}\mathcal{L} &= \mathrm{tr}_c\left(\overline{q}_{Li} i\gamma^\mu D_\mu q_{Li}\right) + \mathrm{tr}_c\left(\overline{q}_{Ri} i\gamma^\mu D_\mu q_{Ri}\right) \\ &\quad + \mathrm{tr}_c\left(\overline{q}_{Li}\gamma^\mu l_{\mu ij} q_{Lj}\right) + \mathrm{tr}_c\left(\overline{q}_{Ri}\gamma^\mu r_{\mu ij} q_{Rj}\right) - \mathrm{tr}_c\left(\overline{q}_{Ri}\mathcal{M}_{ij} q_{Lj}\right) - \mathrm{tr}_c\left(\overline{q}_{Li}\mathcal{M}^\dagger_{ij} q_{Rj}\right) \\ D_\mu q &= \partial_\mu q - iG_\mu q + iqG_\mu, \qquad i,j = 1,2,\ldots,n_f\end{aligned}$$

- The lightest baryon is diquark, with the same mass of pionic Goldstone Boson.
- Transfer the left hand quark to right hand anti-quark:

$$\tilde{q}_{Ri} \equiv C\overline{q}^T_{Li} \qquad C = i\gamma^2\gamma^0$$

- The Lagrangian can be rewritten as

$$\begin{aligned}\mathcal{L} &= \mathrm{tr}_c\left(\overline{\hat{q}} i\gamma^\mu D_\mu \hat{q}\right) + \mathrm{tr}_c\left(\overline{\hat{q}}\gamma^\mu \hat{V}_\mu \hat{q}_j\right) - \frac{1}{2}\mathrm{tr}_c\left(\overline{\hat{q}} C\hat{\mathcal{M}}\overline{\hat{q}}^T\right) - \frac{1}{2}\mathrm{tr}_c\left(\hat{q}^T C\hat{\mathcal{M}}^\dagger \hat{q}\right) \\ \hat{q} &= \begin{pmatrix} q_R \\ \tilde{q}_R \end{pmatrix}, \qquad \hat{V}_\mu = \begin{pmatrix} r_\mu & 0 \\ 0 & -l^T_\mu \end{pmatrix}, \qquad \hat{\mathcal{M}} = \begin{pmatrix} 0 & \mathcal{M} \\ \mathcal{M}^T & 0 \end{pmatrix}\end{aligned}$$

- The actual flavor symmetry is $SU(2n_f)$!
- The vacuum condensate $\langle \mathrm{tr}_c\left(\bar{q}q\right)\rangle = \langle \mathrm{tr}_c\left(\hat{q}^T C J_S \hat{q}\right)\rangle \neq 0$ spontaneously breaks the $SU(2n_f)$ down to $SO(2n_f)$.

$$J_S = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

Quarks live in pseudo-real representation.

- The Lagrangian with external source terms

$$\begin{aligned}\mathcal{L} &= \overline{q}_{Li} i\gamma^\mu D_\mu q_{Li} + \overline{q}_{Ri} i\gamma^\mu D_\mu q_{Ri} \\ &\quad + \overline{q}_{Li}\gamma^\mu l_{\mu ij} q_{Lj} + \overline{q}_{Ri}\gamma^\mu r_{\mu ij} q_{Rj} - \overline{q}_{Ri}\mathcal{M}_{ij} q_{Lj} - \overline{q}_{Li}\mathcal{M}^\dagger_{ij} q_{Rj} \\ D_\mu &= \partial_\mu q - iG_\mu q, \qquad i,j = 1,2,\ldots,n_f\end{aligned}$$

- The lightest baryon is diquark(boson), with the same mass of pionic Goldstone Boson.
- Transfer the left hand quark to right hand anti-quark:

$$\tilde{q}_{R\alpha i} = \epsilon_{\alpha\beta} C \overline{q}^T_{L\beta i} \qquad C = i\gamma^2\gamma^0$$

- The Lagrangian can be rewritten as

$$\begin{aligned}\mathcal{L} &= \overline{\hat{q}} i\gamma^\mu D_\mu \hat{q} + \overline{\hat{q}}\gamma^\mu \hat{V}_\mu \hat{q}_{Lj} - \frac{1}{2}\overline{\hat{q}}_\alpha C\epsilon_{\alpha\beta}\hat{\mathcal{M}}\overline{\hat{q}}^T_\beta - \frac{1}{2}\hat{q}_\alpha\epsilon_{\alpha\beta} C\hat{\mathcal{M}}^\dagger \hat{q}_\beta \\ \hat{q} &= \begin{pmatrix} q_R \\ \tilde{q}_R \end{pmatrix}, \qquad \hat{V}_\mu = \begin{pmatrix} r_\mu & 0 \\ 0 & -l^T_\mu \end{pmatrix}, \qquad \hat{\mathcal{M}} = \begin{pmatrix} 0 & -\mathcal{M} \\ \mathcal{M}^T & 0 \end{pmatrix}\end{aligned}$$

- The actual flavor symmetry is $SU(2n_f)$!
- The vacuum condensate $\langle \overline{q}q \rangle = \langle \hat{q}_\alpha \epsilon_{\alpha\beta} C J_A \hat{q}_\beta \rangle \neq 0$ spontaneously breaks the $SU(2n_f)$ down to $Sp(2n_f)$.

$$J_A = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$$

Broken Generator and Goldstone Bosons

Nonlinear realized Goldstone Boson in different effective theories:

- Complex representation: $SU(n_f) \times SU(n_f)/SU(n_f)$.

$$U = u\mathbb{I}u, \qquad u = \exp\left(\frac{i}{\sqrt{2}F_0}\sum_{a=1}^{N_g}\pi^a T^a\right), \quad N_g = N_f^2 - 1$$

N_f: Number of quarks(flavour)

N_g: Number of pseudo-Goldstone Boson

- Real representation: $SU(2n_f)/SO(2n_f)$.

$$U = uJ_S u^T, \qquad u = \exp\left(\frac{i}{\sqrt{2}F_0}\sum_{a=1}^{N_g}\pi^a T^a\right), \quad N_g = n_f(2n_f + 1) - 1$$

- Pseudo-real representation: $SU(2n_f)/Sp(2n_f)$.

$$U = uJ_A u^T, \qquad u = \exp\left(\frac{i}{\sqrt{2}F_0}\sum_{a=1}^{N_g}\pi^a T^a\right), \quad N_g = n_f(2n_f - 1) - 1$$

The Lagrangian

- Using the general method of CCWZ, we write the three different theories in the same form of Lagrangian, similar to Chiral Perturbation Theory.
- The Leading order Lagrangian

$$\mathcal{L}_2 = \frac{F_0^2}{4}\langle D_\mu U^\dagger D^\mu U + \chi U^\dagger + U\chi^\dagger\rangle$$

- The covariant derivative D_μ and mass parameter χ is different for three different cases.

	complex	real	pseudo-real
D_μ	$\partial_\mu U - ir_\mu U + iUl_\mu$	$\partial_\mu U - iV_\mu U - iU(J_S V_\mu^T J_S)$	$\partial_\mu U - iV_\mu U - iU(J_A V_\mu^T J_A)$
χ	m_0^2	$m_0^2 J_S$	$m_0^2 J_A$

Lagrangian at Next-to-Leading Order

- The p^4 EFT Lagrangian for three different QCD-like theories is also the same:

$$\begin{aligned}\mathcal{L}_4 &= L_0 \mathrm{Tr}\left[D_\mu U (D_\nu U)^\dagger D^\mu U (D^\nu U)^\dagger\right] \\ &+ L_1 \left\{\mathrm{Tr}[D_\mu U (D^\mu U)^\dagger]\right\}^2 + L_2 \mathrm{Tr}\left[D_\mu U (D_\nu U)^\dagger\right] \mathrm{Tr}\left[D^\mu U (D^\nu U)^\dagger\right] \\ &+ L_3 \mathrm{Tr}\left[D_\mu U (D^\mu U)^\dagger D_\nu U (D^\nu U)^\dagger\right] + L_4 \mathrm{Tr}\left[D_\mu U (D^\mu U)^\dagger\right] \mathrm{Tr}\left(\chi U^\dagger + U \chi^\dagger\right) \\ &+ L_5 \mathrm{Tr}\left[D_\mu U (D^\mu U)^\dagger (\chi U^\dagger + U \chi^\dagger)\right] + L_6 \left[\mathrm{Tr}\left(\chi U^\dagger + U \chi^\dagger\right)\right]^2 \\ &+ L_7 \left[\mathrm{Tr}\left(\chi U^\dagger - U \chi^\dagger\right)\right]^2 + L_8 \mathrm{Tr}\left(U \chi^\dagger U \chi^\dagger + \chi U^\dagger \chi U^\dagger\right) \\ &- i L_9 \mathrm{Tr}\left[f^R_{\mu\nu} D^\mu U (D^\nu U)^\dagger + f^L_{\mu\nu} (D^\mu U)^\dagger D^\nu U\right] + L_{10} \mathrm{Tr}\left(U f^L_{\mu\nu} U^\dagger f_R^{\mu\nu}\right) \\ &+ H_1 \mathrm{Tr}\left(f^R_{\mu\nu} f_R^{\mu\nu} + f^L_{\mu\nu} f_L^{\mu\nu}\right) + H_2 \mathrm{Tr}\left(\chi\chi^\dagger\right).\end{aligned}$$

- The devergence structure of low energy coupling constants in $\overline{MS}$ scheme:

$$\begin{aligned} L_i &= (c\mu)^{d-4}\left[\frac{\Gamma_i}{16\pi^2(d-4)} + L_i^r(\mu)\right] \sim \mathcal{O}(10^{-2}) \\ H_i &= (c\mu)^{d-4}\left[\frac{\Gamma_i'}{16\pi^2(d-4)} + H_i^r(\mu)\right] \sim \mathcal{O}(10^{-2}) \end{aligned}$$

Lagrangian at Next-to-Leading Order

The coefficients Γ_i for the three cases that are needed to absorb the divergences at NLO. The last two lines correspond to the terms with H_1 and H_2

i	Complex $SU(N)_L \times SU(n_f)_R/SU(n_f)$	Real $SU(2n_f)/SO(2n_f)$	Pseudo-Real $SU(2n_f)/Sp(2n_f)$
0	$n_f/48$	$(n_f+4)/48$	$(n_f-4)/48$
1	$1/16$	$1/32$	$1/32$
2	$1/8$	$1/16$	$1/16$
3	$n_f/24$	$(n_f-2)/24$	$(n_f+2)/24$
4	$1/8$	$1/16$	$1/16$
5	$n_f/8$	$n_f/8$	$n_f/8$
6	$(n_f^2+2)/(16n_f^2)$	$(n_f^2+1)/(32n_f^2)$	$(n_f^2+1)/(32n_f^2)$
7	0	0	0
8	$(n_f^2-4)/(16n_f)$	$(n_f^2+n_f-2)/(16n_f)$	$(n_f^2-n_f-2)/(16n_f)$
9	$n_f/12$	$(n_f+1)/12$	$(n_f-1)/12$
10	$-n_f/12$	$-(n_f+1)/12$	$-(n_f-1)/12$
1'	$-n_f/24$	$-(n_f+1)/24$	$-(n_f-1)/24$
2'	$(n_f^2-4)/(8n_f)$	$(n_f^2+n_f-2)/(8n_f)$	$(n_f^2-n_f-2)/(8n_f)$

- Complex (ChPT): *J. Gasser and H. Leutwyler, Annals Phys.* **158** *(1984) 142.*
- Pseudo-Real (Two color): small disagreements with *K. Splittorff, et al, Nucl. Phys. B* **620** *(2002) 290 [arXiv:hep-ph/0108040].*
- Real (Adjoint QCD): *J. Bijnens, J. Lu, JHEP* **0911** *(2009) 116 [arXiv:0910.5424].*

Lagrangian at NNLO for Real and Pesudo-real case

Real and Pseudo-real

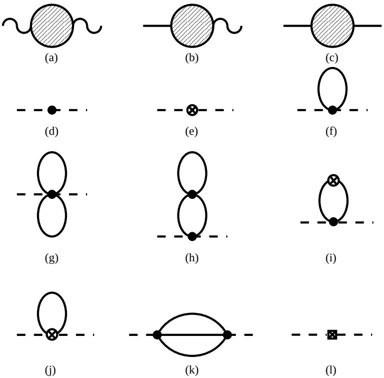
- Their p^6 Lagrangian should have same form of ChPT. Whether it is the minimum form is not known yet.
- The divergence structure form at p^6 for "real" and "pseudo-real" case is the same except the coefficients $\Gamma_i^{(L)}$, $\Gamma_i^{(1)}$ and $\Gamma_i^{(2)}$ they are unknown coefficients.
- The related NNLO coupling constants k_i can be combined as a single unknown parameters for each calculated quantity. The nonlocal divergence should be canceled in the calculation.

Renormalization, the cancellation of divergence.

- The one loop divergence of $\mathcal{L}_2$ is absorbed by the bare coefficient of $\mathcal{L}_4$: L_i
- The two loop divergence of $\mathcal{L}_2$ and one loop divergence of $\mathcal{L}_4$ are absorbed by the bare coefficient of $\mathcal{L}_6$: K_i.

The calculation of important physical quantities up to next-to-next-to leading order:

- Mass of meson
- Quark vacuum condensate
- Decay constant of meson



The set of diagrams contributing to the 1PI quantities.

●: p^2 vertex,
$\times$: p^4 vertex,
$\boxtimes$: p^6 vertex.

$SU(2)$ Case: $\pi\pi$ Scattering

J. Bijnens, J. Lu, JHEP **1103** (2011) 028 [arXiv:1102.0172].

- The amplitude

$$M_{\pi\pi}(s,t,u) = \delta^{ab}\delta^{cd}A(s,t,u) + \delta^{ac}\delta^{bd}A(t,u,s) + \delta^{ad}\delta^{bc}A(u,s,t)\,.$$

- The isospin channels

$$\begin{aligned} T^0(s,t,u) &= 3A(s,t,u) + A(t,s,u) + A(u,t,s),\\ T^1(s,t,u) &= A(t,s,u) - A(u,s,t),\\ T^2(s,t,u) &= A(t,s,u) + A(u,s,t) \end{aligned}$$

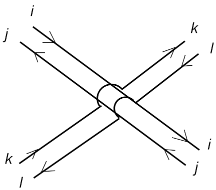
$$M_{\pi\pi}(s,t,u) = \sum_{I=0,1,2} T^I(s,t,u)P_I.$$

General Formula

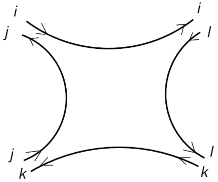
The general amplitude of meson-meson scattering $a + b \to c + d$

$$\langle \phi^c(p_c)\phi^d(p_d)|\phi^a(p_a)\phi^b(p_b)\rangle = M(s,t,u)$$

$$\begin{aligned} M(s,t,u) = & [tr_F(X^aX^bX^cX^d) + tr_F(X^aX^dX^cX^b)]B(s,t,u) \\ & +[tr_F(X^aX^cX^dX^b) + tr_F(X^aX^bX^dX^c)]B(t,u,s) \\ & +[tr_F(X^aX^dX^bX^c) + tr_F(X^aX^cX^bX^d)]B(u,s,t) \\ & +\delta^{ab}\delta^{cd}C(s,t,u) + \delta^{ac}\delta^{bd}C(t,u,s) + \delta^{ad}\delta^{bc}C(u,s,t) \end{aligned}$$



$(\phi_a)^i_j(\phi_a)^j_i(\phi_c)^k_l(\phi_d)^l_k \cdot C$



$(\phi_a)^i_j(\phi_b)^j_k(\phi_c)^k_l(\phi_d)^l_i \cdot B$

Representations and Channels

- Meson is the component of adjoint representations.
- The direct product of two adjoint representations can be written in term of direct sum irreducible representations.
    - Examples for $n_f = 2, 3$:

$$SU(2) : 3 \otimes 3 = 1 \oplus 3 \oplus 5$$
$$SU(3) : 8 \otimes 8 = 1 \oplus 8 \oplus 8' \oplus 10 \oplus \overline{10} \oplus 27$$

    - For general n_f flavour in Complex representation: $SU(n)$

$$\mathbf{Adj.} \otimes \mathbf{Adj.} = \mathbf{R_I} \oplus \mathbf{R_S} \oplus \mathbf{R_A} \oplus + \mathbf{R_S^A} \oplus \mathbf{R_A^S} \oplus \mathbf{R_A^A} \oplus \mathbf{R_S^S}$$

$\Rightarrow$ 7 channels.

- Real representation(adjoint QCD): $SO(2n)$

$$Sym. \otimes Sym. = R_I \oplus R_A \oplus R_S \oplus R_{FS} \oplus R_{MA} \oplus R_{MS} \,.$$

$\Rightarrow$ 6 channels

- Pseudo-real representation(two color QCD): $Sp(2n)$

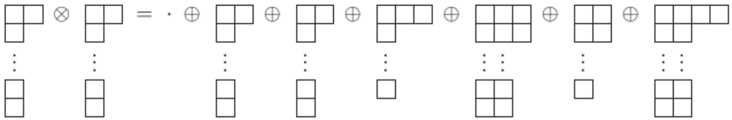
$$Asym. \otimes Asym. = R_I \oplus R_A \oplus R_S \oplus R_{FA} \oplus R_{MA} \oplus R_{MS} \,.$$

$\Rightarrow$ 6 channels.

Representations and Channels: Complex case

Two ways to write down the irreducible representations: tensor method or Young diagrams.

- The scattering channels/irreducible representations(QCD):
  - $R_I : {(\phi_a)_j}^i {(\phi_b)_i}^j$
  - $R_S : (\phi_a)_j(\phi_b)^i + (\phi_b)_j(\phi_a)^i$
  - $R_A : (\phi_a)_j(\phi_b)^i - (\phi_b)_j(\phi_a)^i$
  - $R_S^A : {(\phi_a)_j}^i {(\phi_b)_l}^k - (i \leftrightarrow k) + (j \leftrightarrow l) - (i \leftrightarrow k, j \leftrightarrow l)$
  - $R_A^S : {(\phi_a)_j}^i {(\phi_b)_l}^k + (i \leftrightarrow k) - (j \leftrightarrow l) + (i \leftrightarrow k, j \leftrightarrow l)$
  - $R_A^A : {(\phi_a)_j}^i {(\phi_b)_l}^k - (i \leftrightarrow k) - (j \leftrightarrow l) + (i \leftrightarrow k, j \leftrightarrow l)$
  - $R_S^S : {(\phi_a)_j}^i {(\phi_b)_l}^k + (i \leftrightarrow k) + (j \leftrightarrow l) + (i \leftrightarrow k, j \leftrightarrow l)$
- Young diagrams



Scattering amplitude

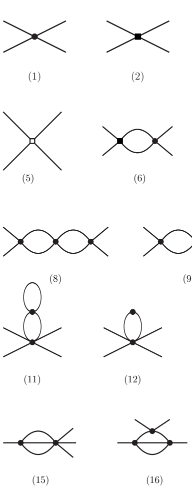
Scattering amplitude is defined as

$$T_r = \langle R_r | M(s,t,u) | R_r \rangle$$

The scattering amplitude for complex case

$$\begin{aligned}
T_I &= 2\left(n - \frac{1}{n}\right)[B(s,t,u) + B(t,u,s)] - \frac{2}{n}B(u,s,t) \\
&\quad + (n^2 - 1)C(s,t,u) + C(t,u,s) + C(u,s,t) \ , \\
T_S &= \left(n - \frac{4}{n}\right)[B(s,t,u) + B(t,u,s)] - \frac{4}{n}B(u,s,t) \\
&\quad + C(t,u,s) + C(u,s,t) \ , \\
T_A &= n[-B(s,t,u) + B(t,u,s)] + C(t,u,s) - C(u,s,t) \ , \\
T_{SA} &= C(t,u,s) - C(u,s,t) \ , \\
T_{AS} &= C(t,u,s) - C(u,s,t) \ , \\
T_{SS} &= 2B(u,s,t) + C(t,u,s) + C(u,s,t) \ , \\
T_{AA} &= -2B(u,s,t) + C(t,u,s) + C(u,s,t) \ .
\end{aligned}$$

Fynman Diagrams



The full analytic result of two loop vertex diagram in the equal mass case is presented.

Two-point functions

J. Bijnens, J. Lu, JHEP **1201** (2012) 082[arXiv:1111.1886].
Definition of two-point functions:

$$\begin{aligned}
\Pi^a_{V\mu\nu}(q) &\equiv i\int d^4x\, e^{iq\cdot x}\,\langle 0|T(V^a_\mu(x)V^a_\nu(0))^\dagger|0\rangle \\
&= (q_\mu q_\nu - q^2 g_{\mu\nu})\Pi^{(1)}_{Va}(q^2) + q_\mu q_\nu \Pi^{(0)}_{Va}(q^2)\,,
\end{aligned}$$

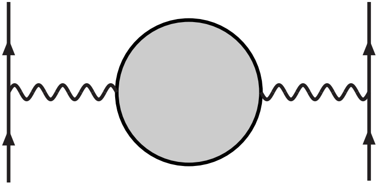
$$\begin{aligned}
\Pi^a_{A\mu\nu}(q) &\equiv i\int d^4x\, e^{iq\cdot x}\,\langle 0|T(A^a_\mu(x)A^a_\nu(0))^\dagger|0\rangle \\
&= (q_\mu q_\nu - q^2 g_{\mu\nu})\Pi^{(1)}_{Aa}(q^2) + q_\mu q_\nu \Pi^{(0)}_{Aa}(q^2)\,,
\end{aligned}$$

$$\Pi^a_S(q) \equiv i\int d^4x\, e^{iq\cdot x}\,\langle 0|T(S^a(x)S^a(0))^\dagger|0\rangle\,,$$

$$\Pi^a_P(q) \equiv i\int d^4x\, e^{iq\cdot x}\,\langle 0|T(P^a(x)P^a(0))^\dagger|0\rangle$$

OBLIQUE CORRECTION AND S-PARAMETER

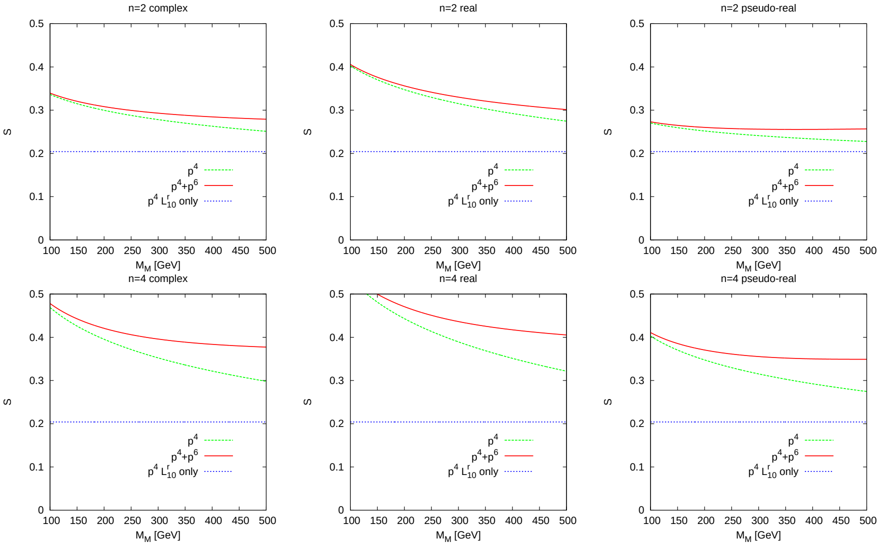
- Three types of one-loop correction to LEP process $e^+ + e^- \to q + \bar{q}$: vacuum polarization corrections, vertex corrections, and box corrections.
  The vertex corrections and and box corrections are suppressed by factor of m_f^2/m_Z^2.
- The one-loop oblique correction



- The S parameter can be written as

$$
\begin{aligned}
S &= -2\pi\left[\Pi'_{VV}(0) - \Pi'_{AA}(0)\right] \\
&= 2\pi \frac{d}{dq^2}\left(q^2\Pi^{(1)}_{VV} - q^2\Pi^{(1)}_{AA}\right)_{q^2=0} .
\end{aligned}
$$

Oblique correction and S-parameter



Summary

- We have written the Effective Field Theory of three QCD-like theories (Complex, Real, Pseudo-real) in an extremely similar way which can simplify the calculation.
- We got the one loop divergence coefficients Γ_i for real and pseuso-real case.
- We calculated the m_π, F_π and $\langle \bar{q}q \rangle$ in the $SU(N_f)$ Chiral Perturbation Theory and other two QCD-like Theories for equal mass case up to NNLO.
- We studied the general meson-meson scattering for three QCD-like theories, analysed the amplitude structure, calculated scattering length up to NNLO.
- We calculated the V-V, A-A, P-P, S-S two-point functions up to NNLO, obtained the formula of S-parameter in those QCD-like theories.
- All the three theories maybe useful for the study of Technicolor Theory and Finite Baryon Density, especially for people who are working in Lattice.

Welcome to Valencia!