$B_{s,d} \to \ell^+\ell^-$ Decays in the Aligned 2HDM

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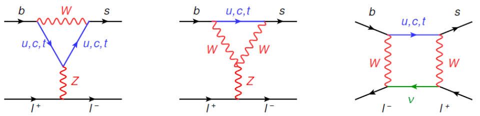
Outline

Motivation to study the $B_{s,d} \to \ell^+\ell^-$ decays:

- Three important features of purely leptonic B-meson decays:
  - ✓ forbidden at tree level, can only proceed through higher-order loop diagrams $\Rightarrow$ responsible for the extremely rare nature of these decays;
  - ✓ due to the $(V-A)$ nature of SM weak interactions, suffer from a helicity-suppression factor $m_\ell/m_b \Rightarrow$ very sensitive to non-SM scalar and pseudoscalar interactions;
  - ✓ characterized by a purely leptonic final state $\Rightarrow$ theoretically very clean, with the only hadronic uncertainty coming from the decay constants f_{B_s} and f_{B_d};

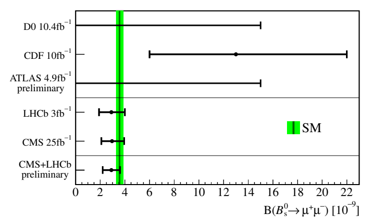


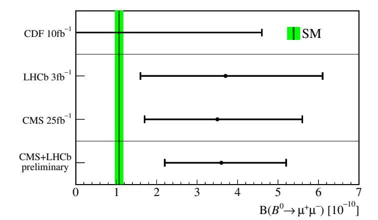
- testing the SM at loop-level and probing physics beyond the SM, especially of models with a non-standard Higgs sector; [Buras and Girrbach, arXiv:1306.3775 [hep-ph]]

The Exp. status of $B_{s,d} \to \ell^+\ell^-$ decays:

- These decay channels are among the highest priorities in heavy flavour physics @ LHCb, CMS and ATLAS; [LHCb collaboration, arXiv:1208.3355 [hep-ex]]

- $B_{s,d} \to \mu^+\mu^-$: especially interesting (easily tagged); first evidence reported by LHCb in 2012/11; [CMS-PAS-BPH-13-007, LHCb-CONF-2013-012, CERN-LHCb-CONF-2013-012]





$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)_{\text{exp.}} = (2.9 \pm 0.7) \times 10^{-9}\,, \quad \overline{\mathcal{B}}(B_d \to \mu^+\mu^-)_{\text{exp.}} = \left(3.6\,^{+1.6}_{-1.4}\right) \times 10^{-10}$$

- Exp. prospects for the decay $B_s \to \mu^+\mu^-$:
  - ✓ @ LHCb with 50 fb^{-1}: 5% error of the SM; [LHCb collaboration, arXiv:1208.3355 [hep-ex]]
  - ✓ @ CMS with 100 fb^{-1}: 15% error of the SM; [Kai-Feng Chen, talk at KEK FFWS, 2014]

Status of the SM predictions for $B_{s,d} \to \ell^+\ell^-$:

- The currently most precise SM predictions: NLO EW + NNLO QCD; [Bobeth, Gorbahn, Hermann, Misiak, Stamou, and Steinhauser, arXiv:1311.0903 [hep-ph]]

$$\overline{\mathcal{B}}_{B_s\to\mu^+\mu^-} = (3.65 \pm 0.23) \times 10^{-9}, \qquad \overline{\mathcal{B}}_{B_d\to\mu^+\mu^-} = (1.06 \pm 0.09) \times 10^{-10}$$

$$\overline{\mathcal{B}}_{B_s\to e^+e^-} = (8.54 \pm 0.55) \times 10^{-14}, \qquad \overline{\mathcal{B}}_{B_d\to e^+e^-} = (2.48 \pm 0.21) \times 10^{-15}$$

$$\overline{\mathcal{B}}_{B_s\to\tau^+\tau^-} = (7.73 \pm 0.49) \times 10^{-7}, \qquad \overline{\mathcal{B}}_{B_d\to\tau^+\tau^-} = (2.22 \pm 0.19) \times 10^{-8}$$

- A summary of the error budgets for $B_s \to \mu^+\mu^-$ and $B_d \to \mu^+\mu^-$: [Bobeth, Gorbahn, Hermann, Misiak, Stamou, and Steinhauser, arXiv:1311.0903 [hep-ph]]

Error budget	f_{B_q}	CKM	τ_H^q	M_t	α_s	other param.	non-param.	$\sum$
$\overline{\mathcal{B}}_{B_s\to\mu^+\mu^-}$	4.0%	4.3%	1.3%	1.6%	0.1%	$< 0.1\%$	1.5%	6.4%
$\overline{\mathcal{B}}_{B_d\to\mu^+\mu^-}$	4.5%	6.9%	0.5%	1.6%	0.1%	$< 0.1\%$	1.5%	8.5%

$f_{B_s} = 227.7\,(4.5)$ MeV [FLAG, 1310.8555], $|V_{cb}| = 0.0424\,(9)$ [Gambino and Schwanda 1307.4551]

- The theo. accuracy is essential in interpreting the data in terms of the SM or NP.

Overview of the A2HDM: I

- A Higgs-like particle discovered at the LHC $\Rightarrow$ It could just be the SM Higgs? or are there more scalars? [ATLAS, arXiv:1207.7214 [hep-ex]; CMS, arXiv:1207.7235 [hep-ex]]

- 2HDM: the simplest non-trivial extension on the SM Higgs sector; [Branco, Ferreira, Lavoura, Rebelo, Sher and Silva, 1106.0034; Gunion, Haber, Kane and Dawson, Front.Phys.**80**, 1 (2000)]

  ✓ duplicate a complex $SU(2)_L$ Higgs doublet with the same hypercharge $Y = \frac{1}{2}$;

  ✓ featured by five physical Higgs states, especially a charged Higgs boson;

  ✓ rich and viable phenomenologies in collider and low-energy flavour physics;

- In a generic 2HDM: tree-level FCNC interactions $\Rightarrow$ how to avoid them?

  ✓ assuming very large scalar masses and/or very small scalar couplings;

  ✓ in the Type-III 2HDM model: $Y_f \propto \sqrt{m_i m_j}$; [Cheng and Sher, Phys. Rev. **D** 35 (1987) 3484]

  ✓ imposing discrete $\mathcal{Z}_2$ symmetries: only one $\phi_a(x)$ couples to a given $f_R(x)$;

  $\Rightarrow$ the Type-I, II, X and Y 2HDMs; [Glashow and Weinberg, Phys. Rev. D **15** (1977) 1958]

Overview of the A2HDM: II

- A2HDM: the Yukawa matrices are aligned in flavour space for each type of $f_R(x) \Rightarrow$ A more general alternative to avoid FCNC! [Pich and Tuzón, 0908.1554]

- Two Higgs doublets: $\phi_a \; (a = 1, 2)$

$$\langle 0|\phi_a^T(x)|0\rangle = \frac{1}{\sqrt{2}}\left(0, v_a \mathrm{e}^{i\theta_a}\right), \qquad \theta_1 = 0, \qquad \theta \equiv \theta_2 - \theta_1$$

- Higgs basis: perform a global $SU(2)$ transformation in the (ϕ_1, ϕ_2) space, with $\tan\beta \equiv v_2/v_1$; [Davidson and Haber, hep-ph/0504050; Haber and O'Neil, hep-ph/0602242, 1011.6188]

$$\begin{pmatrix} \Phi_1 \\ -\Phi_2 \end{pmatrix} \equiv \begin{bmatrix} \cos\beta & \sin\beta \\ \sin\beta & -\cos\beta \end{bmatrix} \begin{pmatrix} \phi_1 \\ \mathrm{e}^{-i\theta}\phi_2 \end{pmatrix}$$

$$\Phi_1 = \begin{bmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(v + S_1 + iG^0\right) \end{bmatrix}, \qquad \Phi_2 = \begin{bmatrix} H^+ \\ \frac{1}{\sqrt{2}}\left(S_2 + iS_3\right) \end{bmatrix}$$

✓ only Φ_1 gets a nonzero vev; ✓ $G^\pm, G^0$: Goldstone bosons;

✓ Five mass eigenstates: $H^\pm(x)$, $\varphi_i^0(x) = \{h(x), H(x), A(x)\} = \mathcal{R}_{ij}\, S_j(x)$;

Yukawa interactions of A2HDM: I

- The most general Yukawa Lagrangian of the 2HDM:

$$\mathcal{L}_Y = -\left\{ \bar{Q}'_L(\Gamma_1\phi_1 + \Gamma_2\phi_2)\, d'_R + \bar{Q}'_L(\Delta_1\widetilde{\phi}_1 + \Delta_2\widetilde{\phi}_2)\, u'_R + \bar{L}'_L(\Pi_1\phi_1 + \Pi_2\phi_2)\, l'_R \right\} + \text{h.c.}$$

- Moving to the Higgs basis, and after the SSB: $\Downarrow$

$$\mathcal{L}_Y = -\frac{\sqrt{2}}{v} \left\{ \bar{Q}'_L(M'_d\Phi_1 + Y'_d\Phi_2)\, d'_R + \bar{Q}'_L(M'_u\tilde{\Phi}_1 + Y'_u\tilde{\Phi}_2)\, u'_R + \bar{L}'_L(M'_l\Phi_1 + Y'_l\Phi_2)\, l'_R \right\} + \text{h.c}$$

$$M'_d = \frac{1}{\sqrt{2}} \left(v_1\Gamma_1 + v_2\Gamma_2 e^{i\theta} \right), \qquad Y'_d = \frac{1}{\sqrt{2}} \left(v_1\Gamma_2 e^{i\theta} - v_2\Gamma_1 \right)$$

$$M'_u = \frac{1}{\sqrt{2}} \left(v_1\Delta_1 + v_2\Delta_2 e^{-i\theta} \right), \qquad Y'_u = \frac{1}{\sqrt{2}} \left(v_1\Delta_2 e^{-i\theta} - v_2\Delta_1 \right)$$

$$M'_l = \frac{1}{\sqrt{2}} \left(v_1\Pi_1 + v_2\Pi_2 e^{i\theta} \right), \qquad Y'_l = \frac{1}{\sqrt{2}} \left(v_1\Pi_2 e^{i\theta} - v_2\Pi_1 \right)$$

- Note: M'_f and Y'_f are unrelated, cannot be simultaneously diagonalized $\Rightarrow$ with diagonal M_f, Y_f remain non-diagonal, giving rise to tree-level FCNCs.

Yukawa interactions of A2HDM: II

- A2HDM: requiring the alignment in flavour space of the Yukawa matrices

$$\Gamma_2 = \xi_d \, e^{-i\theta} \, \Gamma_1 \,, \qquad \Delta_2 = \xi_u^* \, e^{i\theta} \Delta_1 \,, \qquad \Pi_2 = \xi_l \, e^{-i\theta} \, \Pi_1$$

$$\Downarrow$$

$$Y_{d,l} = \varsigma_{d,l} M_{d,l} \,, \qquad Y_u = \varsigma_u^* M_u \,, \qquad \varsigma_f \equiv \frac{\xi_f - \tan\beta}{1 + \xi_f \tan\beta}$$

- In the mass-eigenstate basis, the Yukawa Lagrangian reads:

$$\mathcal{L}_Y = -\frac{\sqrt{2}}{v} H^+(x) \left\{ \bar{u}(x) \left[\varsigma_d \, V_{\rm CKM} \, M_d \mathcal{P}_R - \varsigma_u \, M_u^\dagger V_{\rm CKM} \mathcal{P}_L \right] d(x) \, + \, \varsigma_l \, \bar{\nu}(x) M_l \mathcal{P}_R l(x) \right\}$$

$$- \, \frac{1}{v} \sum_{\varphi, f} y_f^{\varphi_i^0} \, \varphi_i^0(x) \, \bar{f}(x) \, M_f \mathcal{P}_R f(x) \, + \, \text{h.c.}$$

$$y_{d,l}^{\varphi_i^0} = \mathcal{R}_{i1} + (\mathcal{R}_{i2} + i \, \mathcal{R}_{i3}) \, \varsigma_{d,l} \,, \qquad y_u^{\varphi_i^0} = \mathcal{R}_{i1} + (\mathcal{R}_{i2} - i \, \mathcal{R}_{i3}) \, \varsigma_u^*$$

- The alignment parameters ς_f: can be complex; satisfying the universality among different generations; being scalar-basis independent.

Yukawa interactions of A2HDM: III

$$\mathcal{L}_Y = -\frac{\sqrt{2}}{v} H^+(x) \left\{ \bar{u}(x) \left[\varsigma_d V_{\text{CKM}} M_d \mathcal{P}_R - \varsigma_u M_u^\dagger V_{\text{CKM}} \mathcal{P}_L \right] d(x) + \varsigma_l \bar{\nu}(x) M_l \mathcal{P}_R l(x) \right\}$$

$$- \frac{1}{v} \sum_{\varphi, f} y_f^{\varphi_i^0} \varphi_i^0(x) \bar{f}(x) M_f \mathcal{P}_R f(x) + \text{h.c.}$$

- only three new universal parameters ς_f are introduced;
- all fermionic couplings to scalars are proportional to fermion masses;
- all the neutral-current interactions are diagonal in flavour;
- V_{CKM} is the only source of flavour-changing interactions;
- all leptonic couplings are diagonal in flavour due to the absence of ν_R;
- ς_f: arbitrary complex numbers $\Rightarrow$ new sources of CP violation without tree-level FCNCs;
- The usual $\mathcal{Z}_2$-symmetric models recovered in the limit $\xi_f \to 0, \infty, \tan\beta$;

Model	ς_d	ς_u	ς_l
Type-I	$\cot\beta$	$\cot\beta$	$\cot\beta$
Type-II	$-\tan\beta$	$\cot\beta$	$-\tan\beta$
Type-X	$\cot\beta$	$\cot\beta$	$-\tan\beta$
Type-Y	$-\tan\beta$	$\cot\beta$	$\cot\beta$
Inert	0	0	0

Yukawa interactions of A2HDM: IV

- $Y_{d,l} = \varsigma_{d,l} M_{d,l}$, $Y_u = \varsigma_u^* M_u$: presumably held at some high-energy scale, but are spoiled by radiative corrections induced by scalars; [Pich and Tuzón, 0908.1554]

- Possible FCNC interactions constrained by the flavour symmetries of A2HDM:

$$f_X^i(x) \to \mathrm{e}^{i\alpha_i^{f,X}} f_X^i(x)\,, \qquad V_{ij} \to \mathrm{e}^{i\alpha_i^{u,L}} V_{ij}\, \mathrm{e}^{-i\alpha_j^{d,L}}\,, \qquad M_{f,ij} \to \mathrm{e}^{i\alpha_i^{f,L}} M_{f,ij}\, \mathrm{e}^{-i\alpha_j^{f,R}}$$

$$\Downarrow$$

✓ Leptonic FCNCs are absent to all orders in perturbation theory ($\alpha_i^{\nu,L} = \alpha_i^{l,L}$);

✓ The only allowed local FCNC structures in the quark sector take the form:

$$\bar{u}_L V (M_d M_d^\dagger)^n V^\dagger (M_u M_u^\dagger)^m M_u u_R\,, \qquad \bar{d}_L V^\dagger (M_u M_u^\dagger)^n V (M_d M_d^\dagger)^m M_d d_R$$

✓ V_{CKM} remains the only possible source of flavour-changing transitions;

- Satisfy the popular MFV structure, allow at the same time for new CP-violating phases; [D'Ambrosio, Giudice, Isidori and Strumia, hep-ph/0207036; Buras, Carlucci, Gori, Isidori, 1005.5310]

Yukawa interactions of A2HDM: V

- Local FCNC interactios at one loop: obtained using the RGEs; [Cvetic, Kim and Hwang, hep-ph/9806282; Ferreira, Lavoura and Silva, 1001.2561; Braeuninger, Ibarra and Simonetto, 1005.5706]

$$\mathcal{L}_{\rm FCNC} = \frac{\mathcal{C}}{4\pi^2 v^3}\,(1+\varsigma_u^*\,\varsigma_d)\sum_i \varphi_i^0 \Big\{ (\mathcal{R}_{i2}+i\,\mathcal{R}_{i3})\,(\varsigma_d-\varsigma_u)\,\left[\bar{d}_L\,V^\dagger M_u M_u^\dagger V M_d\,d_R\right]$$

$$-\,(\mathcal{R}_{i2}-i\,\mathcal{R}_{i3})\,(\varsigma_d^*-\varsigma_u^*)\,\left[\bar{u}_L\,V M_d M_d^\dagger V^\dagger M_u\,u_R\right]\Big\} + \text{h.c.}$$

- $\mathcal{C} = \mathcal{C}_R(\mu) + \frac{1}{2}\left\{\frac{2\mu^{D-4}}{D-4} + \gamma_E - \ln(4\pi)\right\}$, $\qquad \mathcal{C}_R(\mu) = \mathcal{C}_R(\mu_0) - \ln(\mu/\mu_0)$

- vanishes in all $\mathcal{Z}_2$ models as it should: $\varsigma_d = \varsigma_u$ (types I, X and inert) or $\varsigma_d = -1/\varsigma_u^*$ (types II and Y);

- suppressed by the factor $m_q m_{q'}^2/(4\pi^2 v^3)$ and the quark-mixing factors $\Rightarrow$ most relevant in the $\bar{s}_L b_R$ and $\bar{c}_L t_R$ operators;

- its tree-level contributions are needed to render finite the contributions of the one-loop Higgs-penguin diagrams to $B_{s,d} \to \ell^+\ell^-$;

The scalar potential of A2HDM: I

- The most general scalar potential takes the form: [Davidson and Haber, hep-ph/0504050; Haber and O'Neil, hep-ph/0602242, 1011.6188; Branco, Ferreira, Lavoura, Rebelo, Sher and Silva, 1106.0034]

$$\begin{aligned} V &= \mu_1 \left(\Phi_1^\dagger \Phi_1\right) + \mu_2 \left(\Phi_2^\dagger \Phi_2\right) + \left[\mu_3 \left(\Phi_1^\dagger \Phi_2\right) + \mu_3^* \left(\Phi_2^\dagger \Phi_1\right)\right] \\ &+ \lambda_1 \left(\Phi_1^\dagger \Phi_1\right)^2 + \lambda_2 \left(\Phi_2^\dagger \Phi_2\right)^2 + \lambda_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right) \\ &+ \left[\left(\lambda_5\, \Phi_1^\dagger \Phi_2 + \lambda_6\, \Phi_1^\dagger \Phi_1 + \lambda_7\, \Phi_2^\dagger \Phi_2\right)\left(\Phi_1^\dagger \Phi_2\right) + \text{h.c.}\right] \end{aligned}$$

- Hermiticity of V: all parameters to be real except for μ_3, λ_5, λ_6 and λ_7;

- Minimization conditions: $\mu_1 = -\lambda_1 v^2$, $\mu_3 = -\frac{1}{2}\lambda_6 v^2$;

- Freedom to rephase Φ_2: only the relative phases among λ_5, λ_6 and λ_7 are physical;

$\Downarrow$

- V is characterized by 11 parameters: v, μ_2, $\lambda_{1,2,3,4}$, $|\lambda_{5,6,7}|$, $\arg(\lambda_5\lambda_6^*)$, $\arg(\lambda_5\lambda_7^*)$;

The scalar potential of A2HDM: II

- The mass term: obtained from the quadratic term in V;

$$V_2 = M_{H\pm}^2 H^+H^- + \frac{1}{2}(S_1, S_2, S_3)\,\mathcal{M}\begin{pmatrix} S_1 \\ S_2 \\ S_3 \end{pmatrix} = M_{H\pm}^2 H^+H^- + \frac{1}{2}\sum_{i=1}^{3} M_{\varphi_i^0}^2 \left(\varphi_i^0\right)^2$$

- The charged scalar mass: $M_{H\pm}^2 = \mu_2 + \frac{1}{2}\lambda_3 v^2$

- The neutral scalar masses: $\mathcal{R}\,\mathcal{M}\,\mathcal{R}^T = \mathrm{diag}\left(M_h^2, M_H^2, M_A^2\right)$, $\quad \varphi_i^0 = \mathcal{R}_{ij} S_j$

$$\mathcal{M} = \begin{pmatrix} 2\lambda_1 v^2 & v^2\,\lambda_6^{\mathrm{R}} & -v^2\,\lambda_6^{\mathrm{I}} \\ v^2\,\lambda_6^{\mathrm{R}} & M_{H\pm}^2 + v^2\left(\frac{\lambda_4}{2} + \lambda_5^{\mathrm{R}}\right) & -v^2\,\lambda_5^{\mathrm{I}} \\ -v^2\,\lambda_6^{\mathrm{I}} & -v^2\,\lambda_5^{\mathrm{I}} & M_{H\pm}^2 + v^2\left(\frac{\lambda_4}{2} - \lambda_5^{\mathrm{R}}\right) \end{pmatrix}$$

- In the CP-conserving limit: $A = S_3$, $\quad \begin{pmatrix} h \\ H \end{pmatrix} = \begin{bmatrix} \cos\tilde{\alpha} & \sin\tilde{\alpha} \\ -\sin\tilde{\alpha} & \cos\tilde{\alpha} \end{bmatrix} \begin{pmatrix} S_1 \\ S_2 \end{pmatrix}$

$$M_h^2 = \frac{1}{2}(\Sigma - \Delta)\,, \qquad M_H^2 = \frac{1}{2}(\Sigma + \Delta)\,, \qquad M_A^2 = M_{H\pm}^2 + v^2\left(\frac{\lambda_4}{2} - \lambda_5^{\mathrm{R}}\right)$$

$$\Sigma = M_{H\pm}^2 + v^2\left(2\,\lambda_1 + \frac{\lambda_4}{2} + \lambda_5^{\mathrm{R}}\right), \quad \Delta = \sqrt{\left[M_{H\pm}^2 + v^2\left(-2\,\lambda_1 + \frac{\lambda_4}{2} + \lambda_5^{\mathrm{R}}\right)\right]^2 + 4v^4(\lambda_6^{\mathrm{R}})^2}$$

- The cubic and quartic terms: give rise to the Higgs self interactions.

Phenomenological studies within the A2HDM:

- The $h(126)$ boson data with the A2HDM:

  [Celis, Ilisie and Pich, "LHC constraints on two-Higgs doublet models", 1302.4022; "Towards a general analysis of LHC data within two-Higgs-doublet models", 1310.7941]

- The charged Higgs effects in low-energy flavour physics:

  [Jung, Pich and Tuzón, "Charged-Higgs phenomenology in the A2HDM", 1006.0470; "The $B \to X_s\gamma$ Rate and CP Asymmetry within the A2HDM", 1011.5154; Jung, Li and Pich, "Exclusive radiative B-meson decays within the A2HDM", 1208.1251]

- The charged Higgs effects on (semi-)taunic decays ($R(D)$ and $R(D*)$):

  [Celis, Jung, Li and Pich, "Sensitivity to charged scalars in $\boldsymbol{B \to D^{(*)}\tau\nu_\tau}$ and $\boldsymbol{B \to \tau\nu_\tau}$ decays", 1210.8443]

- EDM within the A2HDM:

  [Jung and Pich, "Electric Dipole Moments in Two-Higgs-Doublet Models", 1308.6283]

- **$\boldsymbol{B_{s,d} \to \ell^+\ell^-}$ decays within the A2HDM:**

  [Li, Lu and Pich, "$B_{s,d} \to \ell^+\ell^-$ Decays in the Aligned Two-Higgs-Doublet Model", 1404.5865]

The effective weak Hamiltonian:

- The effective weak Hamiltonian: obtained after decoupling the heavy degree of freedom (t, $W^\pm$, Z, $H^\pm$, h, H, A); [Buras, Fleischer, Girrbach and Knegjens, 1303.3820]

$$\mathcal{H}_{\rm eff} = -\frac{G_F\,\alpha}{\sqrt{2}\pi s_W^2}\left[V_{tb}V_{tq}^*\sum_i^{10,S,P}\left(C_i\,\mathcal{O}_i + C_i'\,\mathcal{O}_i'\right) + \text{h.c.}\right]$$

$$\mathcal{O}_{10} = (\bar{q}\gamma_\mu P_L b)\,(\bar{\ell}\gamma^\mu\gamma_5\ell)\,, \qquad \mathcal{O}_{10}' = (\bar{q}\gamma_\mu P_R b)\,(\bar{\ell}\gamma^\mu\gamma_5\ell)$$

$$\mathcal{O}_S = \frac{m_\ell m_b}{M_W^2}\,(\bar{q}P_R b)\,(\bar{\ell}\ell)\,, \qquad \mathcal{O}_S' = \frac{m_\ell m_b}{M_W^2}\,(\bar{q}P_L b)\,(\bar{\ell}\ell)\,,$$

$$\mathcal{O}_P = \frac{m_\ell m_b}{M_W^2}\,(\bar{q}P_R b)\,(\bar{\ell}\gamma_5\ell)\,, \qquad \mathcal{O}_P' = \frac{m_\ell m_b}{M_W^2}\,(\bar{q}P_L b)\,(\bar{\ell}\gamma_5\ell)$$

- $\mathcal{O}_i'$: proportional to the light-quark mass $m_q \ll m_b$ and can be neglected;
- Operators with $\bar{\ell}\gamma_\mu\ell$: vanish when contracted with the B-meson momentum p_B^μ;
- Tensor operators: have no contributions due to $\langle 0|\bar{q}\sigma_{\mu\nu}b|\bar{B}_q^0(p)\rangle = 0$;
- only three operators $\mathcal{O}_{10}$, $\mathcal{O}_S$ and $\mathcal{O}_P$ survive in our approximation;
- The WCs of these operators receive no renormalization due to QCD corrections;

The computational method:

- C_{10}, C_S, C_P: require equality of 1PI Green functions calculated in the full and in the effective theory; [Buchalla, Buras and Lautenbacher, hep-ph/9512380; Buras, hep-ph/9806471]

- In the full theory: need to evaluate various box, penguin and self-energy diagrams;

- Heavy-Mass-Expansion: the Feynman integrands expanded in external momenta $l^2 << M^2$ before performing the loop integration; [Smirnov, hep-th/9412063]

$$\frac{1}{(k+l)^2 - M^2} = \frac{1}{k^2 - M^2}\left[1 - \frac{l^2 + 2(k\cdot l)}{k^2 - M^2} + \frac{4(k\cdot l)^2}{(k^2 - M^2)^2}\right] + \mathcal{O}(l^4/M^4)$$

- Partial-Fraction-Decomposition: the final resulting Feynman integrals are of the massive tadpole type; [Bobeth, Misiak and Urban, hep-ph/9910220]

$$\frac{1}{(q^2 - m_1^2)(q^2 - m_2^2)} = \frac{1}{m_1^2 - m_2^2}\left[\frac{1}{q^2 - m_1^2} - \frac{1}{q^2 - m_2^2}\right]$$

- The limit $m_{u,c} \to 0$ and $V_{uq}^* V_{ub} + V_{cq}^* V_{cb} + V_{tq}^* V_{tb} = 0$ should also be exploited;

Results for the Wilson coefficients:

- The final results for the WCs: SM plus A2HDM

$$C_{10} = C_{10}^{\text{SM}} + C_{10}^{\text{Z penguin, A2HDM}}$$

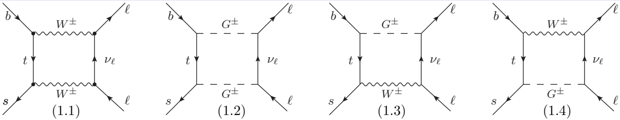
$$C_S = C_S^{\text{box, SM}} + C_S^{\text{box, A2HDM}} + C_S^{\varphi_i^0\text{, A2HDM}}$$

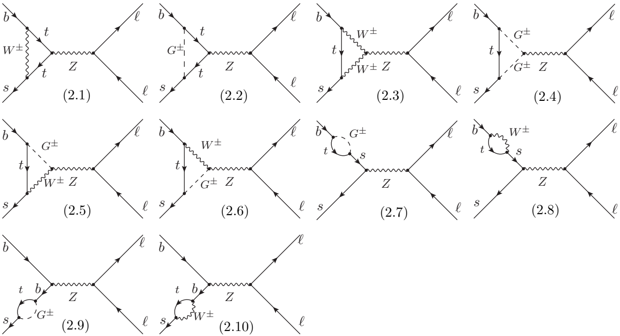
$$C_P = C_P^{\text{box, SM}} + C_P^{\text{Z penguin, SM}} + C_P^{\text{GB penguin, SM}} + C_P^{\text{box, A2HDM}}$$
$$+ C_P^{\text{Z penguin, A2HDM}} + C_P^{\text{GB penguin, A2HDM}} + C_P^{\varphi_i^0\text{, A2HDM}}$$

- The whole calculations are performed in both the Feynman ($\xi = 1$) and the unitary ($\xi = \infty$) gauges, to check the gauge independence;
- While the box and penguin diagrams are separately gauge dependent, their sum is indeed gauge independent; [Buchalla, Buras and Harlander, Nucl. Phys. B **349** (1991) 1; Botella and Lim, Phys. Rev. Lett. **56** (1986) 1651]
- Need not to consider the photonic penguin diagrams in the decays $B_{s,d} \to \ell^+\ell^-$, due to the vector nature of the EM current;

Wilson coefficients in the SM: I

- C_{10} in the SM: generated from the W-box and Z-penguin diagrams; [Inami and Lim, '81; Buchalla and Buras, '99; Bobeth, Gorbahn, Hermann, Misiak, Stamou and Steinhauser, 1311.0903]



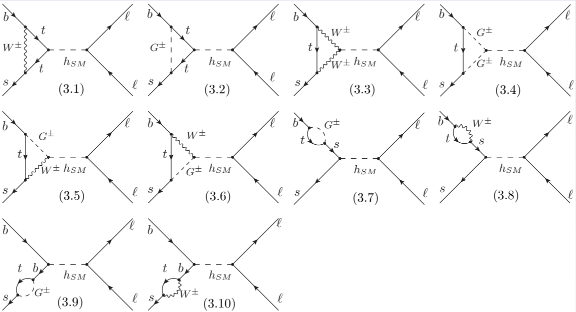


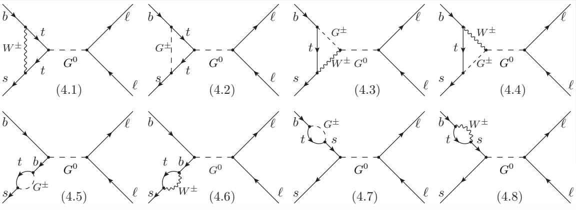
$$C_{10}^{\rm SM} = -\eta_Y^{\rm EW}\,\eta_Y^{\rm QCD}\,Y_0(x_t)$$

- $\eta_Y^{\rm EW} = 0.977$: the N-LO EW matching corrections and QED RG running; [Bobeth, Gorbahn and Stamou, 1311.1348]
- $\eta_Y^{\rm QCD} = 1.010$: the N-LO and NNLO QCD corrections; [Hermann, Misiak and Steinhauser, 1311.1347]

Wilson coefficients in the SM: II

- C_S and C_P in the SM: generated from the W-box, Z-penguin, Higgs-penguin and Goldstone-penguin diagrams; [Botella and Lim, '86; Grzadkowski and Krawczyk, '83; Krawczyk, '89]





$$C_S^{\rm SM} = C_{S,\,\rm Feynman}^{\rm box,\,SM} + C_{S,\rm Feynman}^{\rm h\ penguin,\,SM}$$

$$= C_{S,\,\rm Unitary}^{\rm box,\,SM} + C_{S,\,\rm Unitary}^{\rm h\ penguin,\,SM}$$

$$C_P^{\rm SM} = C_{P,\rm Unitary}^{\rm box,\,SM} + C_{P,\,\rm Unitary}^{\rm Z\ penguin,\,SM}$$

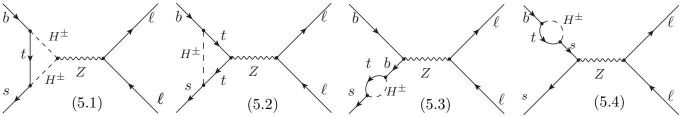
$$= C_{P,\,\rm Feynman}^{\rm box,\,SM} + C_{P,\rm Feynman}^{\rm Z\ penguin,\,SM}$$

$$+ C_{P,\,\rm Feynman}^{\rm GB\ penguin,\,SM}$$

- The linear term in external momenta should be taken into account in the HME;
- The Higgs penguin diagram is by itself gauge-dependent, and is cancelled by that of the W-box;

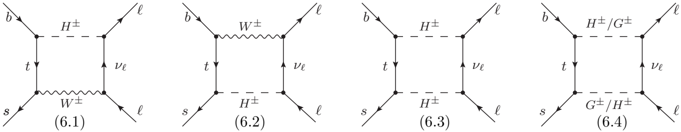
Wilson coefficients in the A2HDM: I

- C_{10} in the A2HDM: generated only from the Z-penguin diagrams involving $H^{\pm}$;



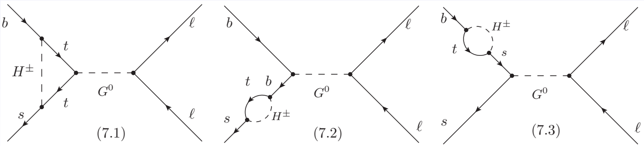
$$C_{10}^{\text{A2HDM}} = |\varsigma_u|^2 f_0(x_t, x_{H^+})$$

- Box diagrams in the A2HDM: contribute only to C_S and C_P;



$$C_{S,P}^{\text{box}} = \varsigma_\ell \, \varsigma_u^* f_1(x_t, x_{H^+}) + \varsigma_u \, \varsigma_\ell^* f_2(x_t, x_{H^+}) + \varsigma_d \, \varsigma_\ell^* f_3(x_t, x_{H^+})$$

- Goldstone-penguin diagrams: contribute only to C_P;

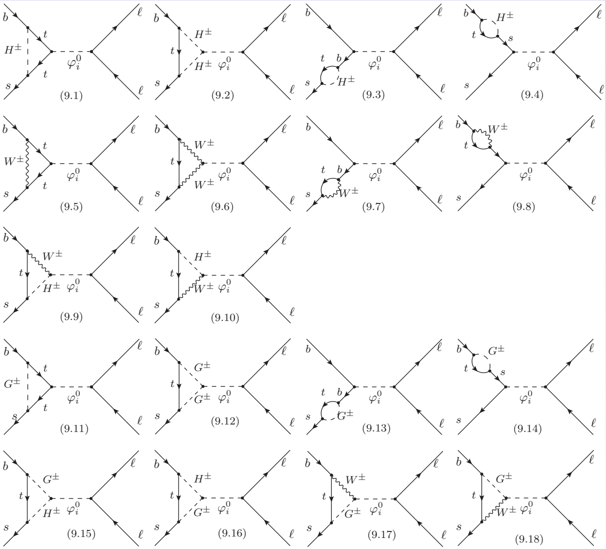


$$C_P^{\text{Z penguin}} = |\varsigma_u|^2 f_4(x_t, x_{H^+}) + \varsigma_d \, \varsigma_u^* f_5(x_t, x_{H^+})$$

$$C_P^{\text{GB penguin}} = |\varsigma_u|^2 f_6(x_t, x_{H^+})$$

Wilson coefficients in the A2HDM: II

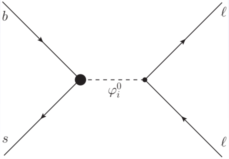
- Higgs-penguin diagrams: one-loop penguin plus tree-level contribution from $\mathcal{L}_{\rm FCNC}$



$$C_S^{\varphi_i^0,\,\rm A2HDM} = \sum_{\varphi_i^0} {\rm Re}(y_\ell^{\varphi_i^0})\,\hat{C}^{\varphi_i^0}$$

$$C_P^{\varphi_i^0,\,\rm A2HDM} = i\sum_{\varphi_i^0} {\rm Im}(y_\ell^{\varphi_i^0})\,\hat{C}^{\varphi_i^0}$$

- (9.1), (9.3), (9.11) and (9.13) in Feynman and 9.1, 9.3–9.7 in unitary gauge still divergent even after GIM;
- the renormalization of $\mathcal{C}$ cancels this divergenc;



Wilson coefficients in the A2HDM: III

- The total scalar-exchange contribution: tree-level plus one-loop penguin;

$$\hat{C}^{\varphi_i^0} = x_t \left\{ \frac{1}{2x_{\varphi_i^0}} (\varsigma_u - \varsigma_d)(1+\varsigma_u^*\varsigma_d)(\mathcal{R}_{i2}+i\mathcal{R}_{i3})\,\mathcal{C}_R(M_W) + \frac{v^2}{M^2_{\varphi_i^0}}\lambda^{\varphi_i^0}_{H^+H^-}\, g_0\,(x_t, x_{H^+}, \varsigma_u, \varsigma_d) \right.$$
$$\left. + \sum_{j=1}^{3} \mathcal{R}_{ij}\,\xi_j \left[\frac{1}{2x_{\varphi_i^0}} g_j^{(a)}(x_t, x_{H^+}, \varsigma_u, \varsigma_d) + g_j^{(b)}(x_t, x_{H^+}, \varsigma_u, \varsigma_d) \right] \right\}$$

$$\lambda^{\varphi_i^0}_{H^+H^-} = \lambda_3\mathcal{R}_{i1} + \lambda_7^R\mathcal{R}_{i2} - \lambda_7^I\mathcal{R}_{i3}, \qquad \xi_1 = \xi_2 = 1, \qquad \xi_3 = i$$

- The orthogonality relation: $\sum_{i=1}^{3} y_\ell^{\varphi_i^0}\,\mathcal{R}_{ij} = \delta_{j1} + (\delta_{j2} + i\,\delta_{j3})\,\varsigma_\ell$

$$C_S^{\varphi_i^0,\,\text{A2HDM}}\Big|_{g^{(b)}} = x_t\left[g_1^{(b)} + \text{Re}(\varsigma_\ell)\,g_2^{(b)} - i\,\text{Im}(\varsigma_\ell)\,g_3^{(b)}\right], \qquad C_P^{\varphi_i^0,\,\text{A2HDM}}\Big|_{g^{(b)}} = x_t\left[i\,\text{Im}(\varsigma_\ell)\,g_2^{(b)} - \text{Re}(\varsigma_\ell)\,g_3^{(b)}\right]$$

- Checking the gauge independence: ⇓

$$C_{S,\,\text{Unitary}}^{\text{box, SM}} - C_{S,\,\text{Feynman}}^{\text{box, SM}} = x_t\, g_1^{(b)}, \qquad C_{S,\,\text{Unitary}}^{\text{box, A2HDM}} - C_{S,\,\text{Feynman}}^{\text{box, A2HDM}} = x_t\left[\text{Re}(\varsigma_\ell)\,g_2^{(b)} - i\,\text{Im}(\varsigma_\ell)\,g_3^{(b)}\right]$$

$$C_{P,\,\text{Unitary}}^{\text{box, A2HDM}} - C_{P,\,\text{Feynman}}^{\text{box, A2HDM}} = x_t\left[i\,\text{Im}(\varsigma_\ell)\,g_2^{(b)} - \text{Re}(\varsigma_\ell)\,g_3^{(b)}\right]$$

The branching ratios of $B_{s,d} \to \ell^+\ell^-$ decays: I

- The B_q meson: due to the pseudoscalar nature, only need the decay constants f_{B_q}

$$\langle 0|\bar{q}\,\gamma_\mu\gamma_5\,b|\bar{B}_q(p)\rangle = if_{B_q}p_\mu\,, \qquad \langle 0|\bar{q}\,\gamma_5\,b|\bar{B}_q(p)\rangle = -if_{B_q}\frac{M_{B_q}^2}{m_b+m_q}$$

- The branching ratio of $B_{s,d} \to \ell^+\ell^-$ decays: neglecting the $B_q - \bar{B}_q$ mixings;

$$\begin{aligned}\mathcal{B}(B_q^0 \to \ell^+\ell^-) &= \frac{\tau_{B_q}\,G_F^4\,M_W^4}{8\pi^5}\left|V_{tb}\,V_{tq}^*\,C_{10}^{\rm SM}\right|^2 f_{B_q}^2 M_{B_q} m_\ell^2\sqrt{1-\frac{4m_\ell^2}{M_{B_q}^2}}\left[|P|^2+|S|^2\right],\\ &= \mathcal{B}(B_q^0 \to \ell^+\ell^-)_{\rm SM}\left[|P|^2+|S|^2\right]\end{aligned}$$

$$P \equiv \frac{C_{10}}{C_{10}^{\rm SM}} + \frac{M_{B_q}^2}{2M_W^2}\left(\frac{m_b}{m_b+m_q}\right)\frac{C_P - C_P^{\rm SM}}{C_{10}^{\rm SM}}\,, \qquad S \equiv \sqrt{1-\frac{4m_\ell^2}{M_{B_q}^2}}\,\frac{M_{B_q}^2}{2M_W^2}\left(\frac{m_b}{m_b+m_q}\right)\frac{C_S - C_S^{\rm SM}}{C_{10}^{\rm SM}}$$

- Note: $\mathcal{O}_S$ and $\mathcal{O}_P$ are suppressed by $M_{B_q}^2/M_W^2$ with respect to that from $\mathcal{O}_{10}$;

The branching ratios of $B_{s,d} \to \ell^+\ell^-$ decays: II

- The averaged time-integrated branching ratio: considering the $B_q - \bar{B}_q$ mixings;
  [De Bruyn, Fleischer, Knegjens etal, 1204.1735; 1204.1737; Buras, Fleischer, Girrbach, Knegjens, 1303.3820]

$$\overline{\mathcal{B}}(B_q^0 \to \ell^+\ell^-) = \left[\frac{1 + \mathcal{A}_{\Delta\Gamma}^{\ell\ell}\, y_q}{1 - y_q^2}\right] \mathcal{B}(B_q^0 \to \ell^+\ell^-)\,, \qquad y_q = \frac{\Delta\Gamma_q}{2\Gamma_q}$$

$\mathcal{A}_{\Delta\Gamma}^{\ell\ell}$: a time-dependent observable;

- The averaged time-integrated branching ratio in the SM: $\mathcal{A}_{\Delta\Gamma}^{\ell\ell} = 1$;

$$\overline{\mathcal{B}}(B_q^0 \to \ell^+\ell^-)_{\rm SM} = \frac{G_F^4\, M_W^4}{8\pi^5\, \Gamma_H^q} \left|V_{tb}\, V_{tq}^*\, C_{10}^{\rm SM}\right|^2 f_{B_q}^2 M_{B_q} m_\ell^2 \sqrt{1 - \frac{4m_\ell^2}{M_{B_q}^2}}$$

- In the absence of beyond-SM sources of CP violation:

$$\begin{aligned}\overline{\mathcal{B}}(B_q^0 \to \ell^+\ell^-) &= \overline{\mathcal{B}}(B_q^0 \to \ell^+\ell^-)_{\rm SM} \left[|P|^2 + \left(1 - \frac{\Delta\Gamma_q}{\Gamma_L^q}\right)|S|^2\right] \\ &= \overline{R}_{q\ell}\, \overline{\mathcal{B}}(B_q^0 \to \ell^+\ell^-)_{\rm SM}\,, \qquad \overline{R}_{q\ell} = \left[|P|^2 + \left(1 - \frac{\Delta\Gamma_q}{\Gamma_L^q}\right)|S|^2\right]\end{aligned}$$

- $\overline{R}_{s\mu} = 0.79 \pm 0.20\,, \overline{R}_{d\mu} = 3.38^{+1.53}_{-1.35}$: used as constraints on model parameters;

Choice of the model parameters:

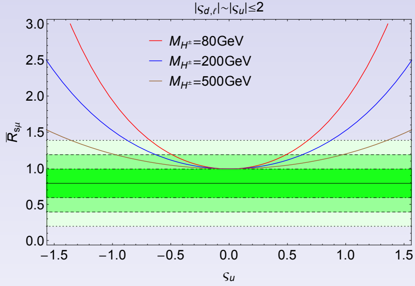
- There are totally 10 free parameters: CP-conserving, h is assumed to be $h(126)$

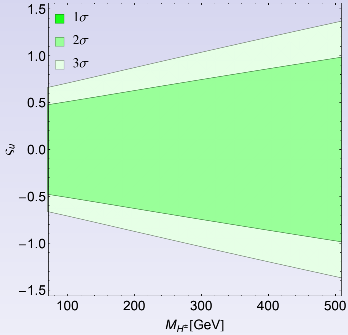
$$\varsigma_{u,d,\ell}\,, \qquad (M_H, M_A, M_{H^\pm})\,, \qquad \tilde{\alpha}\,, \qquad (\lambda_3, \lambda_7)\,, \qquad C_R(M_W)$$

  - ✓ the mixing angle $\tilde{\alpha}$: constrained at $|\cos\tilde{\alpha}| > 0.90$ (68% CL) through a global fit to the latest LHC and Tevatron data for the $h(126)$ boson, very close to the SM limit; [Celis, Ilisie and Pich, 1302.4022; 1310.7941]
  - ✓ $|\lambda_{3,7}| \lesssim 8\pi$: to assure the validity of perturbative unitarity of the scalar-scalar scattering amplitudes; [Branco, Ferreira, Lavoura, Rebelo, Sher and Silva, 1106.0034; Gunion, Haber, Kane and Dawson, Front. Phys. **80**, 1 (2000)]
  - ✓ Neutral scalar masses: $M_H \geq M_h, \quad M_H \in [130, 500]$ GeV, $\quad M_A \in [80, 500]$ GeV;
  - ✓ The charged-Higgs mass: $M_{H^\pm} \in [80, 500]$ GeV, requiring $|\varsigma_u| \leq 2$ constrained by $Z \to \bar{b}b$, $b \to s\gamma$, $B^0_{s,d} - \bar{B}^0_{s,d}$ mixings, and $h(126)$ decays; [Celis, Ilisie and Pich, 1302.4022; 1310.7941; Jung, Pich and Tuzón, 1006.0470; Jung, Li and Pich, 1208.1251]
  - ✓ ς_d, ς_ℓ, $C_R(M_W)$: no strong constraints at the moment;
- $\overline{R}_{s\mu}$ less sensitive to $\lambda_{3,7}$, $\tilde{\alpha}$, $C_R(M_W)$: $\quad \lambda_{3,7} = 1$, $\cos\tilde{\alpha} = 0.95$, $C_R(M_W) = 0$;

Case I: small ς_d and ς_ℓ

- $\varsigma_{d,\ell}$ of the same size as ς_u: $C_{S,P}$ negligible, C_{10}^{A2HDM} involves only ς_u and $M_{H^\pm}$

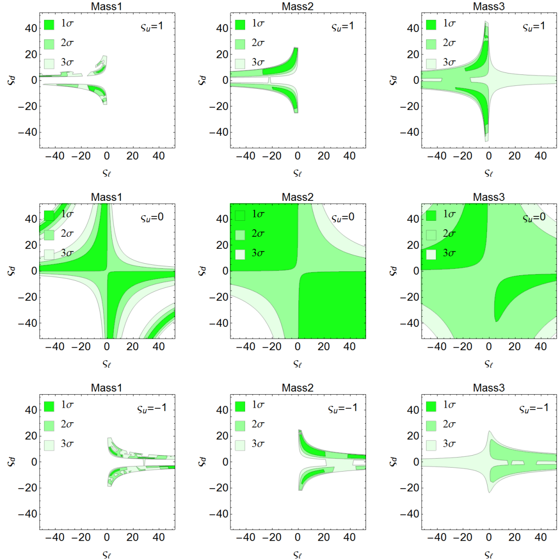




- With C_S and C_P ignored, the ratio $\bar{R}_{s\mu}$ puts strong constraints on the parameter ς_u;
- For $M_{H^\pm} = 80\ (500)$ GeV, $|\varsigma_u| \le 0.52\ (1.03)$ at 95% CL; stronger than from R_b;
- For large $M_{H^\pm}$, the constraint becomes weaker as $\lim_{x_{H^+}\to\infty} C_{10}^{\text{A2HDM}} = 0$;

Case II: large ς_d and ς_ℓ

- When $\varsigma_{d,\ell} \in [-50, 50]$, C_S and C_P can induce a significant enhancement:



Mass1 :

$M_{H^\pm} = M_A = 80\ \text{GeV}, M_H = 130\ \text{GeV}$

Mass2 :

$M_{H^\pm} = M_A = M_H = 200\ \text{GeV}$

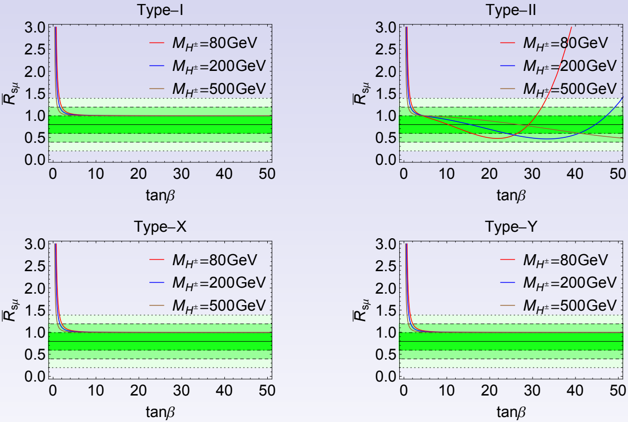
Mass3 :

$M_{H^\pm} = M_A = M_H = 500\ \text{GeV}$

- Regions with large $\varsigma_{d,\ell}$ are already excluded, especially when they have the same sign;
- The impact of ς_u are significant, due to the factor $\varsigma_d^2 \varsigma_u^*$ in $g_2^{(a)}$ and $g_3^{(a)}$;

The discrete $\mathcal{Z}_2$-symmetric models:

- The discrete $\mathcal{Z}_2$-symmetric models: particular cases of the CP-conserving A2HDM



- type-I, type-X and type-Y are almost indistinguishable;
- $\tan\beta \geq 1.5$ at 95% CL under constraint from $\bar{R}_{s\mu}$;
- For type-II model, an enhancement of $\bar{R}_{s\mu}$ is still possible in the large $\tan\beta$ region;

- For the inert model: no mixing between h and H, and H, A and $H^{\pm}$ decouple from the fermions: $\cos\tilde{\alpha} = 1$, $\lambda_6 = \lambda_7 = 0$, $\varsigma_f = 0$. The couplings of h to fermions and vector bosons identical to the SM ones. Therefore, $\bar{R}_{s\mu}^{\text{inert}} = 1$.

Conclusion:

- $B_{s,d} \to \ell^+\ell^-$ decays are analyzed within the general framework of the A2HDM;
- A complete one-loop calculation of SD WCs C_{10}, C_S and C_P from various box and penguin diagrams are calculated, in both the Feynman and the unitary gauges;
- With the current data on $\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)$, investigated the impact of various model parameters on the branching ratios and the phenomenological constraints imposed by the present data;
- The resulting information about the model parameters will be crucial for the model building and is complementary to the collider physics;
- Next steps: to analyze the impact of new CP-violating phases on the time-dependent $B_{s,d} \to \ell^+\ell^-$ decay amplitude; A detailed global analysis under the collider, low-energy flavour-conserving and flavour-violating processes;

谢 谢 大 家!