Flavor (non)diagonal CP phase and the Higgs physics

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May 17, 2014

Content:

- Flavor nondiagonal and diagonal CP violation
- Strong CP problem in the SM
- Spontaneous CP violation and its difficulty in FCNC
- What type of Yukawa we should have if solving strong CP problem without PQ mechanism
- Spontaneous generation of flavor nondiagonal CP phase, the KM CP phase
- Radiative corrections
- Summary

So far CP violation has been observed

- in $K - \bar{K}$ oscillation and Kaon decay
- in $B - \bar{B}$ oscillation and B decay

These are all CP violation associated with flavor conversion of quarks.

CP violation is not observed in flavor diagonal observables

- neutron EDM $\lesssim 10^{-26}$ e cm
- electron EDM $\lesssim 10^{-26}$ e cm

This is understood by the KM theory of CP violation in the SM
A CP violating phase occurs in a special unitary matrix responsible for mixing of quark generations

$$K = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (1)$$

The contribution to neutron EDM by CP phase in the CKM matrix should appear at at least (third) fourth order in perturbation theory and should be $\lesssim 10^{-31} - 10^{-32}$ e cm.

Another CP violation phase, θ, is allowed by the gauge invariance:

$$\Delta\mathcal{L} = \frac{\alpha_s}{8\pi}\theta G^{\mu\nu}\tilde{G}_{\mu\nu} = \frac{\alpha_s}{16\pi}\theta\varepsilon^{\mu\nu\rho\sigma}G_{\mu\nu}G_{\rho\sigma} \tag{2}$$

This term breaks *CP* symmetry.

It gives a contribution to the electric dipole moment, e.g.

$$\text{neutron EDM} \approx e\theta\lambda/m_N^2 \approx \theta \times 10^{-16} \text{ e cm} \tag{3}$$

$\lambda = m_u m_d m_s/(m_u m_d + m_u m_s + m_d m_s)$

So $\theta \lesssim 10^{-9} - 10^{-10}$ according to measurement of EDM.

The problem of strong CP is that we can not simply set θ to zero

- In order to get complex CKM matrix, the Yukawa couplings in the SM have to be complex

$$\bar{Q}Y^u\phi u_R + \bar{Q}Y^d\tilde{\phi}d_R + h.c. \tag{4}$$

and

$$K = V_L^{u\dagger}V^d, \quad Y^u = V_L^u y^u V_R^{u\dagger}, \; Y^d = V_L^d y^d V_R^{d\dagger} \tag{5}$$

where y^u and y_d are diagonal and real, $V_{L,R}^{u,d}$ are unitary matrices.

- So the mass matrix

$$M^u = Y^u v, \quad M^d = Y^d v \tag{6}$$

are in general complex and not hermitian and diagonal. Chiral type transformation, $V_L \neq V_R$, should be involved to go to the mass base

A $U(1)$ part of the chiral transformation will lead to a correction to the θ term

$$\theta \to \quad \theta = \theta_0 + \arg(det(M^u M^d)) = \theta_0 + \arg(det(M^u)) + \arg(det(M^d)) \tag{7}$$

A priori, no principle to guarantee θ small or zero.

Even if θ_0 is set to be zero, a non-zero quantum correction is in general expected

Apparently, the non-observation of flavor diagonal CP violation, such as the neutron EDM, is a mystery.

Suggested solutions to strong CP problem

1) If a quark has mass zero, say up quark, we are allowed to do an arbitrary chiral transformation such that

$$u \to e^{i\alpha\gamma_5} u, \quad m^u \to e^{2i\alpha} m^u = 0 \tag{8}$$

θ can be removed, but current algebra study does not favor(rules out) this scenario

2) The bare up quark mass is zero, its current mass is generated due to instanton correction through chiral anomaly. Whether this mechanism of generating up quark mass can work is still an open question.

3) The Peccei-Quinn mechanism which makes use of instanton potential to minimize the scalar potential in spontaneous breaking of an extra symmetry. It automatically selects a vacuum which has $\theta = 0$. So far no evidence of the predicted axion has been found.

4) Spontaneous CP violation with CP symmetry enforced at high energy scale. θ_0 is set to zero by CP symmetry. It's still difficult to have zero correction to θ. Moreover, several Higgs doublets are needed and in general Flavor Changing Neutral Current occurs at tree level.

It is very difficult to find a solution avoiding flavor diagonal CP while keeping flavor nondiagonal CP with reasonable FCNC

This situation is related to the fact that we do not have a theory of Higgs and its couplings.

Spontaneous CP violation in general involves more than one Higgs doublets, e.g. ϕ_1 and ϕ_2, which in general can both couple to quarks:

$$\bar{Q} Y_1^u \phi_1 u_R + \bar{Q} Y_2^u \phi_2 u_R + \bar{Q} Y_1^d \tilde{\phi}_1 d_R + \bar{Q} Y_2^d \tilde{\phi}_2 d_R$$

So the mass of quarks are given by

$$M^{u,d} = Y_1^{u,d} v_1 + Y_2^{u,d} v_2$$

Unitary transformations that diagonalize the mass matrices do not necessarily diagonalize the Higgs couplings to quarks. So tree level FCNC could be present and this type of theories encounter difficulty.

Solution towards this problem include

- making each ϕ_i couples to one type and only one type of quarks via a Z_2 symmetry

$$\bar{Q} Y_1^u \phi_1 u_R + \bar{Q} Y_2^d \tilde{\phi}_2 d_R$$

However, this theory does not provide a solution to strong CP.

- making Y_1 and Y_2 diagonalizable simultaneously

$$[Y_1 Y_1^\dagger, Y_2 Y_2^\dagger] = 0 \tag{9}$$

Radiative correction will generate at 1st order flavor changing interaction with Higgs boson.

In this talk I will describe a type of model which can

- avoid strong CP problem
- naturally avoid large tree level FCNC
- explain(or generate) CP phase in CKM

The point is

- The Yukawa coupling of one Higgs, say ϕ_2, be the dominant
- The coupling of the other Higgs provide correction
- Hierarchy of Yukawa couplings is naturally protected in radiative correction, similar to 't Hooft's naturalness

Suppose

$$M^u = Y_1^u v_1 + Y_2^u v_2 = M_1^u + M_2^u, \quad ||M_1^u|| \ll |M_2^u|| \tag{10}$$

Working in the base that M_2^u is diagonalized

$$M_2^u = \mathrm{diag}\{0, m_2, m_3\} \tag{11}$$

and writing

$$M^u = \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} + m_2 & x_{23} \\ x_{31} & x_{32} & x_{33} + m_3 \end{pmatrix},$$

for $|x_{ij}| \ll m_{2,3}$ the matrices V_L and V_R that diagonalize M^u are all close to unit matrix

$$V_L \approx 1, \quad V_R \approx 1 \tag{12}$$

Apparently, the flavor changing couplings of Higgs field to quarks are given by Y_1^u and are all suppressed to be order m_u/v

- The radiative correction to Y_1 from couplings of ϕ_2 will be proportional to elements of Y_1.
- The hierarchy between Y_1 and Y_2 is not affected by the radiative corrections.
- The suppression of FCNC is similarly robust against quantum correction

An example will be detailed below

Zero correction to θ can be easily achieved by taking rank of Y_1 and Y_2 both less than 3.

For example, if Y_1 and M_1 are rank 1:

$$M_1 = \begin{pmatrix} x_{11}^u v_1 & x_{12}^u v_1 & x_{13}^u v_1 \\ a_u x_{11}^u v_1 & a_u x_{12}^u v_1 & a_u x_{13}^u v_1 \\ b_u x_{11}^u v_1 & b_u x_{12}^u v_1 & b_u x_{13}^u v_1 \end{pmatrix}$$

and Y_2 and M_2 are rank 2

$$M_2 = \mathrm{diag}\{0, y_2^u v_2, y_3^u v_2\}$$

We can find

$$det(M^u) = det(M_1 + M_2) = x_{11}^u y_2^u y_3^u v_1 v_2 v_2$$

x and y are all real

Similar expression for M^d

$$M^d = Y_1^d v_1^* + Y_2^d v_2^*$$

gives

$$det(M^d) = x_{11}^d y_2^d y_3^d v_1^* v_2^* v_2^*$$

So

$$det(M^u M^d) = x_{11}^u y_2^u y_3^u x_{11}^d y_2^d y_3^d |v_1|^2 |v_2|^4 \tag{13}$$

It is real and $arg(det(M^u M^d)) = 0$, the correction to θ is zero

After spontaneous CP violation, the QCD θ is still zero at tree level

Radiative correction can generate nonzero θ at one loop

An explicit example can be given as follows

$$Y_1^u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} K_{23}^\dagger \, g^u, \quad Y_2^u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} K_{23}^\dagger \, h^u$$

and

$$Y_1^d = K_{13} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} K_{12} \, g^d, \quad Y_2^d = K_{13} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} K_{12} \, h^d$$

$g^{u,d}$ and $h^{u,d}$ are all diagonal matrices with real eigenvalues.

The CP phase in v_1 and v_2 are subject to rephasing and can be taken in v_1:

$$v_1 = |v_1| e^{-i\delta}, \quad v_2 = |v_2|$$

So we can find that

$$M^u = V_L^u m^u, \quad m^u = y^u v, \quad V_R^u = 1, \; V_L^u = \begin{pmatrix} e^{-i\delta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} K_{23}^\dagger$$

and

$$M^d = V_L^d m^d, \quad m^d = y^d v, \quad V_R^d = 1, \; V_L^d = K_{13} \begin{pmatrix} e^{i\delta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} K_{12}$$

where $g^{u,d} = y^{u,d} v/|v_1|$, $h^u = y^{u,d} v/|v_2|$.

So

$$K = V_L^{u\dagger} V_L^d = K_{23} \begin{pmatrix} e^{i\delta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} K_{13} \begin{pmatrix} e^{i\delta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} K_{12} \tag{14}$$

It is equivalent to the standard paramerization of CKM matrix via a vector-like transformtion

$$u_L \to e^{2i\delta} u_L, u_R \to e^{2i\delta} u_R$$

so

$$K \to K = K_{23} \begin{pmatrix} e^{-i\delta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} K_{13} \begin{pmatrix} e^{i\delta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} K_{12}$$

- It's easy to check

$$det(M^u) = e^{-i\delta} det(m^u), \quad det(M^d) = e^{i\delta} det(m^d)$$

  The correction to θ is zero at tree level
- Elements in $Y_1^{u,d}$ are much smaller than elements in $Y_2^{u,d}$, i.e. with the hierarchy described above
- The example here has no flavor changing coupling of Higgs with quarks at tree level
- There are no CP violating in Higgs couplings before the symmetry breaking
- CP phase in CKM is generated after spontaneous breaking of CP symmetry

Yukawa couplings receive radiative correction

- Corrections by the same couplings do not modify the rank of the coupling matrix (zero eigenvalue still zero)
- The correction by other couplings should be carefully taken into account.

Correction to Y_i^u (neglecting lepton Yukawa):

- $\sum_j Tr[N_c(Y_i^u Y_j^{u\dagger} + Y_j^d Y_i^{d\dagger})] Y_j^u$, correction from $Y_{j\neq i}^u$
- $\sum_j (Y_j^u Y_j^{u\dagger} + Y_j^d Y_j^{d\dagger}) Y_i^u$, no impact on the rank of Y_i^u
- $Y_i^u \sum_j Y_j^{u\dagger} Y_j^u$, $\sum_j Y_j^d Y_i^{d\dagger} Y_j^u$

All corrections to Y_i^u are small, but how small?

To second order

$$det(M^u + \delta M^u) = v_1 v_2^2 g_1^u h_2^u h_3^u - 3\epsilon^2 s_{13} s_{23} c_{23} s_{12} c_{12}$$
$$\times (g_2^d h_2^d - g_1^d h_1^d)[(h_3^u)^2 - (h_2^u)^2] g_1^u h_2^u h_3^u \times v_1^2 v_2$$
$$det(M^d + \delta M^d) = v_1^* (v_2^*)^2 g_1^d h_2^d h_3^d + \frac{1}{2}\epsilon^2 s_{13} s_{23} c_{23} s_{12} c_{12}$$
$$\times (g_2^d h_2^d - g_1^d h_1^d)[(h_3^u)^2 - (h_2^u)^2] h_3^d (s_{12}^2 g_2^d h_1^d + c_{12}^2 g_1^d h_2^d)(v_1^*)^3$$

Term proportional to ϵ vanish

$\epsilon = \frac{1}{16\pi^2} ln(\mu/\Lambda)$, $s_{13} s_{13} c_{13} s_{23} c_{23} \approx 3. \times 10^{-5}$,
$g_2^d h_2^d \sim 10^{-5} - 10^{-6}$ and $g_1^u h_2^u h_3^u \sim 10^{-7}$ for $|v_1|/v \sim 0.1$

Correction to QCD θ, $\ll 10^{-10}$.

- The origins of mass of different generations of fermions are not universal.
- The Higgs coupling to light fermions would be proportional to

$$\frac{m_q}{|v_1|} sin\alpha \tag{15}$$

which could be much larger than the m_q/v if $|v_2| \gg |v_1|$.
- Couplings to light fermion can be much larger in this mechanism
- In particular, the smaller the CP-odd fraction of the Higgs, the larger the coupling to light fermions
- Measurements of CP odd component of Higgs and rare decay are complementary

Summary

We propose a model of multi-Higgs doublets model(the simplest with two)

- CP symmetry is exact and all couplings are real
- CP symmetry is spontaneously broken and CP phase in CKM matrix is generated after the symmetry breaking
- QCD θ and large tree level FCNC are all be naturally avoided

The key point is that

- Yukawa couplings to one Higgs field is the dominant but the matrix is rank less than 3
- It is like a theory with Friedberg-Lee symmetry in which all flavor mixing can be explained while strong θ can be made real by a chiral transformation(arbitrary)
- The coupling to other Higgs field break the global symmetry in the dominant coupling terms

Summary

So far, the Standard Model is almost the Standard Theory, except that there is no theory of the Higgs field and its interaction

Fermion masses are not guaranteed to have a universal origin, as in SM. Clues to the origin of CP might exist in the Higgs coupling to light fermions.

Predictions of the model presented here:
Flavor changing Higgs decay, $H \to \bar{q}'q$, very weak or zero;
Higgs couplings to light fermion(u, d) can be much larger than that suppressed by m_q/v

Measurement of Higgs decay branching ratio to the level of $h^0 \to \mu^+\mu^-$ will be very helpful for understanding the physics of Higgs and the origin of CP violation(baryogenesis?)

Can this be done at Higgs factory?