Renormalization-group improved predictions for Higgs boson production at large p_T

Fa Peng Huang
Peking University

Based on the arXiv:1405.xxxx

with Chong Sheng Li, Jian Wang,Hai Tao Li

TeV Workshop 2014 at Sun Yat-sen University, Guangzhou

May 16, 2013

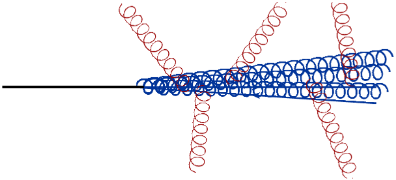
Motivation from Precise Higgs Physics

The 125 GeV Higgs boson has been found at the LHC.

Precise higher order calculations are needed now!

For the Higgs plus one jet process:

- The QCD NLO results are known.
- The two loop helicity amplitude and the total cross section for the gg channel at NNLO are also known.
- In the threshold limit at large p_T, the large logarithms occur, then resummation are needed.



Motivation from New Physics

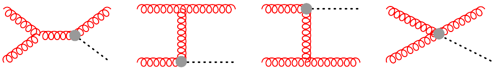
- New physics may modify the coupling of Higgs to the top quark.
- The most direct channel to determine the coupling of Higgs to the top quark is the $t\bar{t}H$ channel.
- However, the current abilities to measure the coupling of Higgs to the top quark through $t\bar{t}H$ process are still weak because of its small production cross section and complicated final states with copious decay products.

Motivation from New Physics

Recently, a complementary method to determine the coupling of Higgs to the top quark has been proposed by investigating the large p_T behavior of Higgs boson in associated production with one jet. (arXiv: 1309.5273, 1312.3317)

Analysis of kinematics and threshold limits

The dominant partonic processes for the Higgs boson and one jet production are $gg \to gH$, $gq \to gH$ and $g\bar{q} \to \bar{q}H$.
For simplicity, we only present the detailed discussions on the subprocess $gg \to gH$ channel as the example.



Analysis of kinematics and threshold limits

The hadronic and partonic threshold variables are defined as

$$S_4 \equiv P_X^2 = s + t + u - M_H^2, \quad (1)$$
$$s_4 \equiv p_X^2 = \hat{s} + \hat{t} + \hat{u} - M_H^2, \quad (2)$$

The hadronic threshold limit is defined as $S_4 \to 0$.
In this limit, the final state radiations and beam remnants are highly suppressed, which leads to final states consisting of a Higgs boson and an inclusive jet, as well as the remaining soft radiations.

Analysis of kinematics and threshold limits

The total cross section is given by

$$\begin{aligned}
\sigma &= \int dx_a \int dx_b \int d\hat{t} \int d\hat{u} f_{i/P_a}(\mu_F, x_a) f_{j/P_b}(\mu_F, x_b) \frac{1}{2\hat{s}} \frac{d\hat{\sigma}_{ij}}{d\hat{t}\, d\hat{u}} \\
&= \int_0^{p^2_{T,max}} dp_T^2 \int_{-y_{max}}^{y_{max}} dy \int_{x_{b,min}}^{1} dx_b \int_0^{s_4^{max}} ds_4 \\
&\quad \frac{1}{2(x_b s + u - M_H^2)} f_{i/P_a}(\mu_F, x_a) f_{j/P_b}(\mu_F, x_b) \frac{d\hat{\sigma}_{ij}}{d\hat{t}\, d\hat{u}} \qquad (3)
\end{aligned}$$

Factorization and Resummation Formalism in SCET

Then we show the main process to get the factorization formula. The first step is writing the the leading power effective operator of $gg \to gH$ in SCET as follows

$$\mathcal{O}_{abc}^{\alpha\beta\gamma}(x; t_1, t_2, t_J) = \mathcal{A}_{1\perp}^{\alpha,a}(x + t_1\bar{n}_1)\,\mathcal{A}_{2\perp}^{\beta,b}(x + t_2\bar{n}_2)\,\mathcal{A}_{J\perp}^{\gamma,c}(x + t_J\bar{n}_J), \tag{4}$$

where $\mathcal{A}_{i\perp}^{\alpha,a}$ is the effective gluon field in the frame of SCET. The corresponding hadronic operator can be written as

$$\mathcal{J}(x) = \int dt_1\, dt_2\, dt_J\, C_{\alpha\beta\gamma}^{abc}(t_1, t_2, t_J)\,\mathcal{O}_{abc}^{\alpha\beta\gamma}(x; t_1, t_2, t_J)\,. \tag{5}$$

Factorization and Resummation Formalism in SCET

The generic expression of the cross section is

$$\begin{aligned} \mathrm{d}\sigma &= \frac{1}{2s}\left(\frac{\alpha_s C_t}{12\pi v}\right)^2 \frac{d^3 p_H}{(2\pi)^3 2E_H} \sum_X (2\pi)^4 \\ &\quad \delta^{(4)}(P_1 + P_2 - p_X - p_H)\left|\langle X\,|\mathcal{J}(0)|N_1(P_1)N_2(P_2)\rangle\right|^2. \end{aligned}$$

Substituting Eqs. (4)– (5) into Eq. (6) and performing Fourier transformation, we get

$$\mathrm{d}\sigma = \frac{1}{2s}\left(\frac{\alpha_s C_t}{12\pi v}\right)^2 \frac{d^3 p_H}{(2\pi)^3 2E_H} \sum_X \widetilde{C}^{abc*}_{\alpha\beta\gamma}\, \widetilde{C}^{def}_{\mu\nu\rho}$$

$$\int \mathrm{d}^4 x\, e^{-i(p_H x)}\, \langle N_1(P_1)\, N_2(P_2)|\, \mathcal{O}^{abc\dagger}_{\alpha\beta\gamma}(x)\, |X\rangle\langle X|\, \mathcal{O}^{def}_{\mu\nu\rho}(0)\, |N_1(P_1)\, N_2(P_2)\rangle$$

Factorization and Resummation Formalism in SCET

Redefining the field to decouple the soft-collinear interaction,

$$\mathcal{O} = \mathcal{O}^s \mathcal{O}^c, \tag{7}$$

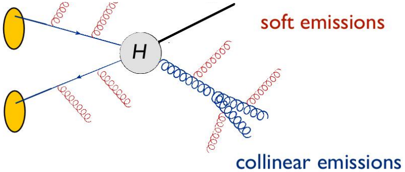
rewrite the squared amplitude in Eq. (6) as

$$\begin{aligned}
&\langle N_1(P_1)\, N_2(P_2)|\, \mathcal{O}^\dagger_{\alpha\beta\gamma}(x)\, |X\rangle\langle X|\, \mathcal{O}_{\mu\nu\rho}(0)\, |N_1(P_1)\, N_2(P_2)\rangle \; = \\
&\left\langle N_1(P_1) \left| \mathcal{A}^\perp_{1\alpha}(x)\mathcal{A}^\perp_{1\mu}(0) \right| N_1(P_1) \right\rangle \times \left\langle N_2(P_2) \left| \mathcal{A}^\perp_{2\beta}(x)\mathcal{A}^\perp_{2\nu}(0) \right| N_2(P_2) \right\rangle \\
&\times \sum_{X_c} \langle 0|\mathcal{A}^\gamma_{J\perp}(x)|X_c\rangle\langle X_c|\mathcal{A}^\rho_{J\perp}(0)|0\rangle \times \sum_{X_s} \langle 0|\mathcal{O}^{s\dagger}_{gg}(x)|X_s\rangle\langle X_s|\mathcal{O}^s_{gg}(0)|0\rangle \,.
\end{aligned} \tag{8}$$

Factorization and Resummation Formalism in SCET

Substituting the definition of the gluon jet , soft and parton distribution functions (PDFs) into Eq. (8), we obtain (up to power corrections) the final factorization formula

$$\begin{aligned}
\sigma &= \int dx_a dx_b d\hat{t} d\hat{u} \frac{1}{2\hat{s}} f_{i/P_a}(x_a,\mu) f_{j/P_b}(x_b,\mu) \frac{d\hat{\sigma}_{ij}^{\mathrm{thres}}}{d\hat{t}d\hat{u}}, \\
\frac{d\hat{\sigma}_{ij}^{\mathrm{thres}}}{d\hat{t}d\hat{u}} &= \frac{1}{8\pi}\frac{1}{\hat{s}} \lambda_{0,ij} H_{ij}(\mu) \\
&\quad \times \int dk^+ \int dp_1^2 \, \mathcal{S}(k^+,\mu) J(p_1^2,\mu) \delta(s_4 - p_1^2 - 2k^+ E_1),
\end{aligned}$$



The Hard, Jet and Soft Functions at NLO

The hard functions are absolute value squared of the Wilson coefficients of the operators, which can be obtained by matching the full theory onto SCET.
It is obtained by subtracting the IR divergences in the $\overline{MS}$ scheme from the UV renormalized amplitudes of the full theory.

The Hard, Jet and Soft Functions at NLO

The hard functions at NLO

$$\begin{aligned}
H_{gg}(\mu_h) &= 1+\frac{\alpha_s(\mu_h)}{4\pi}\Bigg\{-3N_c\ln^2\left(\frac{\mu_h^2}{M_H^2}\right)+c_1^{H,gg}+ \\
&\left[\gamma_{gg}^{H,0}-2N_c\left(\ln\frac{M_H^2}{\hat{s}}+\ln\frac{M_H^2}{-\hat{t}}+\ln\frac{M_H^2}{-\hat{u}}\right)\right]\ln\frac{\mu_h^2}{M_H^2}\Bigg\}, \\
H_{gq}(\mu_h) &= 1+\frac{\alpha_s(\mu_h)}{4\pi}\Bigg[\left(\frac{1}{N_c}-2N_c\right)\ln^2\frac{\mu_h^2}{M_H^2}+c_1^{H,gq}+ \\
&\frac{N_c^2\left(13-6\ln\frac{M_H^2}{\hat{s}}-6\ln\frac{M_H^2}{-\hat{u}}\right)+6\ln\frac{M_H^2}{-\hat{t}}-4N_cn_f+9}{3N_c}\ln\frac{\mu_h^2}{M_H^2}
\end{aligned}$$

The Hard, Jet and Soft Functions at NLO

In our case, the RG equations for hard functions are given by

$$\begin{aligned}
\frac{d}{d\,\ln\mu_h}H_{gg}(\mu_h) &= \left[3\gamma_{\rm cusp}\left(\ln\frac{\hat{s}}{\mu_h^2}+\ln\frac{-\hat{t}}{\mu_h^2}+\ln\frac{-\hat{u}}{\mu_h^2}\right)+2\gamma^H_{gg}\right]H_{gg}(\mu_h),\\
\frac{d}{d\,\ln\mu_h}H_{gq}(\mu_h) &= \left[3\gamma_{\rm cusp}\left(\ln\frac{\hat{s}}{\mu_h^2}+\ln\frac{-\hat{u}}{\mu_h^2}-\frac{1}{9}\ln\frac{-\hat{t}}{\mu_h^2}\right)+2\gamma^H_{gq}\right]H_{gq}(\mu_h)
\end{aligned}$$

The Hard, Jet and Soft Functions at NLO

Solving the RG equations,

$$
\begin{aligned}
H_{gg}(\mu) &= \left(\frac{\alpha_s(\mu_h)}{\alpha_s(\mu)}\right)^3 \exp\left[18S(\mu_h,\mu) - 2a_{gg}^V(\mu_h,\mu)\right] \\
&\quad \left(\frac{\hat{s}\hat{t}\hat{u}}{\mu_h^6}\right)^{-3a_\Gamma(\mu_h,\mu)} H_{gg}(\mu_h), \qquad (10) \\
H_{gq}(\mu) &= \left(\frac{\alpha_s(\mu_h)}{\alpha_s(\mu)}\right)^3 \exp\left[\frac{34}{3}S(\mu_h,\mu) - 2a_{gq}^V(\mu_h,\mu)\right] \\
&\quad \left(\frac{(\hat{s})^9(-\hat{u})^9/(-\hat{t})}{\mu_h^{34}}\right)^{-\frac{1}{3}a_\Gamma(\mu_h,\mu)} H_{gq}(\mu_h), \qquad (11)
\end{aligned}
$$

The Hard, Jet and Soft Functions at NLO

The jet function of gluon $J_g(p^2)$ is defined as

$$\langle 0 | \mathcal{A}^{a\mu}_{J\perp}(x)\mathcal{A}^{b\nu}_{J\perp}(0) | 0 \rangle = (-g^{\mu\nu}_{\perp})\,\delta^{ab}\,g_s^2 \int \frac{\mathrm{d}^4 p}{(2\pi)^3}\,\theta(p^0)\,J_g(p^2)\,e^{-ipx}\,.$$

the RG-improved jet function at an arbitrary scale μ can be obtained

$$J_i(p^2,\mu) = \exp\left[-4C_i S(\mu_j,\mu) + 2a^{J_i}(\mu_j,\mu)\right]\widetilde{j}(\partial_{\eta_j},\mu_j)\frac{1}{p^2}\left(\frac{p^2}{\mu_j^2}\right)^{\eta_j}\frac{e^{-\gamma_E\eta_j}}{\Gamma(\eta_j)},$$

The Hard, Jet and Soft Functions at NLO

The RG-improved soft function can be given as

$$\mathcal{S}(k,\mu) = \exp\left[-4C_{gi}S(\mu_s,\mu) - 2a^S(\mu_s,\mu)\right]\widetilde{s}(\partial_{\eta_s},\mu_s)\frac{1}{k}\left(\frac{k}{\tilde{\mu}_s}\right)^{\eta_s}\frac{e^{-\gamma_E\eta_s}}{\Gamma(\eta_s)},$$

the Laplace transformed soft function $\widetilde{s}(L,\mu)$ at NNLO is given by

$$\begin{aligned}
\widetilde{s}(L,\mu) \;=\;& 1+\frac{\alpha_s}{4\pi}\left(2C_{gi}\Gamma_0 L^2 - 2\gamma_0^S L + c_1^S\right) \\
&+\left(\frac{\alpha_s}{4\pi}\right)^2\Big\{2C_{gi}\Gamma_0^2 L^4 + (-4\gamma_0^S C_{gi}\Gamma_0 - \frac{4\beta_0 C_{gi}\Gamma_0}{3})L^3 \\
&+\left[2C_{gi}\Gamma_1 + 2(\gamma_0^S+\beta_0)\gamma_0^S + 2c_1^S C_{gi}\Gamma_0\right]L^2 \\
&-2\left[\gamma_1^S + (\gamma_0^S+\beta_0)c_1^S\right]L + c_2^S\Big\}.
\end{aligned}$$

Final RG-improved differential cross section

The resummed differential cross section for the Higgs boson and a jet associated production

$$
\begin{aligned}
\frac{d\hat{\sigma}_{ij}^{\rm thres}}{d\hat{t}\,d\hat{u}} = & \sum_{ij} \frac{\lambda_{0,ij}(\mu_h)}{16\pi\hat{s}^2} \\
& \exp\left[\rho_{ij} S(\mu_h,\mu) - 2a_{ij}^{V}(\mu_h,\mu)\right] \left(\frac{\hat{s}\hat{t}\hat{u}}{\mu_h^6}\right)^{-3a_\Gamma(\mu_h,\mu)} H_{ij}(\mu_h) \\
& \exp\left[-4C_i S(\mu_j,\mu) + 2a^{J_i}(\mu_j,\mu)\right] \left(\frac{m_H^2}{\mu_j^2}\right)^{\eta_j} \\
& \exp\left[-4C_{gi} S(\mu_s,\mu) - 2a^{S}(\mu_s,\mu)\right] \left(\frac{m_H^2\sqrt{\hat{s}}}{\mu_s\sqrt{\hat{t}\hat{u}}}\right)^{\eta_s} \\
& \widetilde{j}(\partial_\eta + L_j,\mu_j)\,\widetilde{s}(\partial_\eta + L_s,\mu_s)\frac{1}{s_4}\left(\frac{s_4}{m_H^2}\right)^{\eta} \frac{e^{-\gamma_E\eta}}{\Gamma(\eta)}, \qquad (12)
\end{aligned}
$$

Final RG-improved differential cross section

In order to compare with the fixed-order results, setting $\mu_h = \mu_j = \mu_s = \mu$, we expand the above results up to $\mathcal{O}(\alpha_s^2)$

$$\left(\frac{\lambda_{0,ij}}{16\pi\hat{s}^2}\right)^{-1} \frac{d\hat{\sigma}_{ij}^{\text{thres}}}{d\hat{t}\,d\hat{u}} = \delta(s_4) + \frac{\alpha_s}{4\pi}\left\{A_2 D_2 + A_1 D_1 + A_0\delta(s_4)\right\} + \left(\frac{\alpha_s}{4\pi}\right)^2\left\{B_4 D_4 + B_3 D_3 + B_2 D_2 + B_1 D_1 + B_0\delta(s_4)\right\}, \quad (13)$$

with

$$D_n = \left[\frac{\ln^{n-1}(s_4/M_H^2)}{s_4}\right]_+, \quad (14)$$

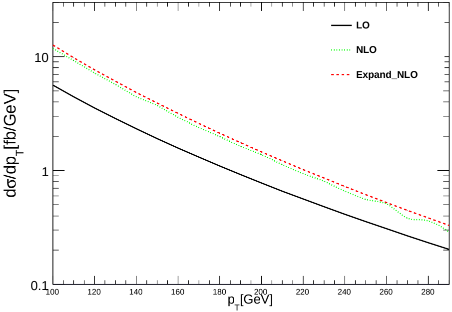
Final RG-improved differential cross section

In order to obtain the best possible precise predictions, we combine our resummed result with the non-singular terms up to NLO in fixed-order perturbative calculations, and the RG-improved differential cross section are given by

$$\frac{d\hat{\sigma}_{\rm Resum}}{d\hat{t}d\hat{u}} = \frac{d\hat{\sigma}^{\rm thres}}{d\hat{t}d\hat{u}} + \left(\frac{d\hat{\sigma}_{\rm NLO}}{d\hat{t}d\hat{u}} - \frac{d\hat{\sigma}^{\rm thres}}{d\hat{t}d\hat{u}}\right)\Big|_{\rm expanded\ to\ NLO}, \quad (15)$$

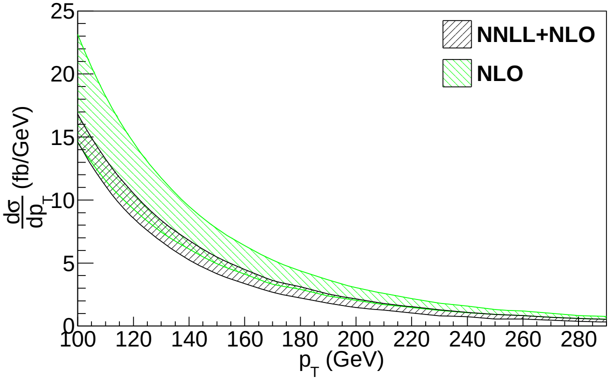
Numerical Results

The NLO cross section is well approximated by the singular terms from resummation when the p_T of Higgs is larger than 100 GeV.



Numerical Results

Choosing scale as $\mu_h = 2.5\sqrt{p_T^2 + M_H^2}$, $\mu_j = 150$ GeV, $\mu_s = 100$ GeV can lead to small scale uncertainty.



Numerical Results

Define the differential K-factor

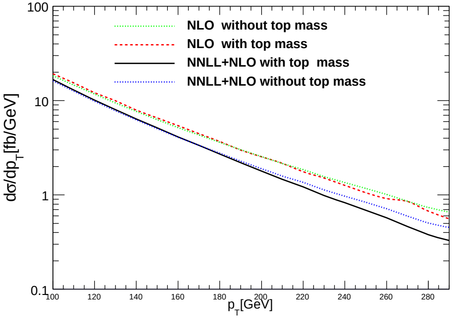
$$K(p_T) = \frac{d\sigma_\infty^{NLO+NNLL}}{d\sigma_\infty^{LO}}, \tag{16}$$

Since the differential K-factor depends weakly on the top quark mass , we can obtain a reliable approximation of higher-order cross section by multiplying the K-factor to the exact top mass dependent LO one.

$$\frac{d\sigma}{dp_T} = \frac{d\sigma_{m_t}^{LO}}{dp_T} K(p_T). \tag{17}$$

Numerical Results

The central value
$(\mu_F = M_H, \mu_h = 2.5\sqrt{p_T^2 + M_H^2}, \mu_j = 150\ \mathrm{GeV}, \mu_{\mathrm{s}} = 100\ \mathrm{GeV})$of four cases for the Higgs production with large p_T at the 8 TeV LHC.



Conclusion

- We have studied the Higgs boson and a jet associated production at large p_T including NNLL+NLO resummation effects in SCET.
- The resummation effects decrease the NLO cross sections by about 11% for $p_T > 100$ GeV, and also reduce the scale uncertainty obviously.
- Moreover, we discuss the top mass effect numerically.
- Any deviation of the Higgs boson's p_T behavior will give hints to the possible modification of the Higgs couplings to the top quark, which will shed light on the new physics.

Thanks!