Higgs Pair production via VBF at QCD NNLO

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- ATLAS and CMS collaborations at the Large Hadron Collider (LHC) have discovered a 126 GeV neutral boson whose properties are compatible with the Standard Model (SM) Higgs boson.
- Measurements of the Higgs self-interactions are necessary to reconstruct the Higgs potential, and determine whether the new particle is the SM Higgs boson or one of an enlarged Higgs sector of new physics.

Higgs pair productions at LHC:

(1) gluon-gluon fusion, $gg \to h^0h^0$, through heavy-quark loop,
(2) vector boson fusion (VBF) $qq' \to V^*V^* \to q''q'''h^0h^0$,
(3) top-quark pair associated Higgs boson pair production $q\bar{q}/gg \to t\bar{t}h^0h^0$,
(4) double Higgs strahlung $qq' \to h^0h^0V$.

Higgs pair production via VBF has the second largest cross section and offers a clean experimental signature of two centrally produced Higgs bosons with two hard jets in the forward/backward rapidity region.

Higgs trilinear self-interactions !

1. 2HDM of type-II

Higgs potential:

$$
\begin{aligned}
V(\Phi_1, \Phi_2) &= m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + \frac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 \\
&\quad + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{1}{2}\lambda_5 \left[(\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \right]
\end{aligned}
$$

$$m_{11}^2,\ m_{22}^2,\ \lambda_i\ (i = 1, ..., 5) \in R$$

- Z_2 discrete symmetry: $\Phi_1 \to \Phi_1, \qquad \Phi_2 \to -\Phi_2$

- Parametrization:

$$\Phi_i = \begin{pmatrix} \phi_i^+ \\ \frac{1}{\sqrt{2}}(v_i + R_i + iI_i) \end{pmatrix}, \qquad (i = 1, 2)$$

- Mass eigenstates:

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = R(\alpha) \begin{pmatrix} R_1 \\ R_2 \end{pmatrix}, \quad \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = R(\beta) \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}, \quad \begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = R(\beta) \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix}$$

G^0 and $G^\pm$ are Nambu-Goldstone bosons and their three degrees of freedom are got "eaten" by the longitudinal components of Z and $W^\pm$ bosons, and induce the masses of weak gauge bosons.

2HDM predicts five scalar particles:

$$h^0, \quad H^0, \quad A^0, \quad H^\pm$$

- Seven independent "physical" input parameters:

$$m_{h^0}, \quad m_{H^0}, \quad m_{A^0}, \quad m_{H^\pm}, \quad \sin\alpha, \quad \tan\beta, \quad v$$

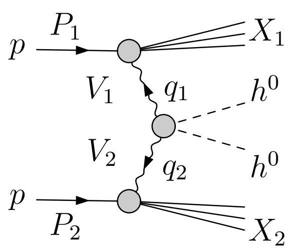
- Higgs couplings:

	WW, ZZ	up-type quarks	down-type quarks, leptons
h^0	$\sin(\beta-\alpha)$	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$
H^0	$\cos(\beta-\alpha)$	$\sin\alpha/\sin\beta$	$\cos\alpha/\cos\beta$
A^0	0	$i\gamma^5\cot\beta$	$i\gamma^5\tan\beta$

(SM limit: $\beta-\alpha=\pi/2$)

2. Structure function approach

- based on the absence or smallness of the QCD interference between the two inclusive final proton remnants,
- a very good approximation for studing the VBF processes at hadron colliders, which is accurate at a precision level well above the typical residual scale and parton distribution function (PDF) uncertainties.



There are two types of topological Feynman diagrams contributing to the VBF Higgs pair production at parton level:

t-channel and u-channel !

The cross section is approximately contributed only by the squared t- and u-channel amplitudes, while their interference contribution is below 0.01%.

Therefore, the VBF Higgs pair production can be viewed as

double deep-inelastic scattering (DIS)

of two (anti)quarks with two virtual weak vector bosons independently emitted from the hadronic initial states fusing into a Higgs boson pair.

Differential cross section:

$$
\begin{aligned}
d\sigma \;=\; & \sum_{(V_1V_2)} \frac{1}{2S} 2G_F^2 M_{V_1}^2 M_{V_2}^2 \frac{1}{\left(Q_1^2 + M_{V_1}^2\right)^2} W_{\mu\nu}^{V_1}(x_1, Q_1^2) \mathcal{M}_{V_1V_2}^{\mu\rho} \\
& \times \frac{1}{\left(Q_2^2 + M_{V_2}^2\right)^2} \mathcal{M}_{V_1V_2}^{*\nu\sigma} W_{\rho\sigma}^{V_2}(x_2, Q_2^2) \\
& \times \frac{d^3\vec{P}_{X_1}}{(2\pi)^3\, 2E_{X_1}} \frac{d^3\vec{P}_{X_2}}{(2\pi)^3\, 2E_{X_2}} ds_1 ds_2 dPS_2(k_1, k_2)\, (2\pi)^4\, \delta^4\left(P_1 + P_2 - P_{X_1} - P_{X_2} - \sum_{j=1,2} k_j\right)
\end{aligned}
$$

DIS hadronic tensor:

$$
W_{\mu\nu}^{V}(x_i, Q_i^2) = \left(-g_{\mu\nu} + \frac{q_{i,\mu} q_{i,\nu}}{q_i^2}\right) F_1^V(x_i, Q_i^2) + \frac{\hat{P}_{i,\mu}\hat{P}_{i,\nu}}{P_i \cdot q_i} F_2^V(x_i, Q_i^2) + i\epsilon_{\mu\nu\alpha\beta} \frac{P_i^\alpha q_i^\beta}{2P_i \cdot q_i} F_3^V(x_i, Q_i^2)
$$

Within the QCD factorization formalism, the structure functions can be expressed as convolutions of the PDFs in proton with the short-distance Wilson coefficient functions.

Structure functions:

- Z-exchange neutral current:

$$F_i^Z(x,Q^2) \;=\; 2f_i(x)\int_0^1 dy\int_0^1 dz\delta(x-yz)\sum_{j=1}^{n_f}\left(v_j^2+a_j^2\right)$$
$$\times\left[q^+_{\mathrm{ns},j}(y,\mu_f)C^+_{i,\mathrm{ns}}(z,Q,\mu_r,\mu_f)+q_{\mathrm{s}}(y,\mu_f)C_{i,\mathrm{q}}(z,Q,\mu_r,\mu_f)+g(y,\mu_f)C_{i,\mathrm{g}}(z,Q,\mu_r,\mu_f)\right]$$
$$F_3^Z(x,Q^2) \;=\; 2f_3(x)\int_0^1 dy\int_0^1 dz\delta(x-yz)\sum_{j=1}^{n_f}2v_ja_j$$
$$\times\left[q^-_{\mathrm{ns},j}(y,\mu_f)C^-_{3,\mathrm{ns}}(z,Q,\mu_r,\mu_f)+q^{\mathrm{v}}_{\mathrm{ns}}(y,\mu_f)C^{\mathrm{v}}_{3,\mathrm{ns}}(z,Q,\mu_r,\mu_f)\right]$$

- W-exchange charge current:

$$F_i^{W^\mp}(x,Q^2) \;=\; f_i(x)\int_0^1 dy\int_0^1 dz\delta(x-yz)\frac{1}{n_f}\sum_{j=1}^{n_f}\left(v_j^2+a_j^2\right)$$
$$\times\left[\pm\,\delta q^-_{\mathrm{ns}}(y,\mu_f)C^-_{i,\mathrm{ns}}(z,Q,\mu_r,\mu_f)+q_{\mathrm{s}}(y,\mu_f)C_{i,\mathrm{q}}(z,Q,\mu_r,\mu_f)+g(y,\mu_f)C_{i,\mathrm{g}}(z,Q,\mu_r,\mu_f)\right]$$
$$F_3^{W^\mp}(x,Q^2) \;=\; f_3(x)\int_0^1 dy\int_0^1 dz\delta(x-yz)\frac{1}{n_f}\sum_{j=1}^{n_f}2v_ja_j$$
$$\times\left[\pm\,\delta q^+_{\mathrm{ns}}(y,\mu_f)C^+_{3,\mathrm{ns}}(z,Q,\mu_r,\mu_f)+q^{\mathrm{v}}_{\mathrm{ns}}(y,\mu_f)C^{\mathrm{v}}_{3,\mathrm{ns}}(z,Q,\mu_r,\mu_f)\right]$$

$f_1(x)=1/2,\ f_2(x)=x,\ f_3(x)=1$

- PDFs:

$$q_{\rm s} = \sum_{i=1}^{n_f} (q_i + \bar{q}_i)\,, \qquad\qquad\qquad\qquad\qquad \text{(singlet)}$$

$$q_{\rm ns}^{\rm v} = \sum_{i=1}^{n_f} (q_i - \bar{q}_i)\,, \quad q_{{\rm ns},ij}^{\pm} = (q_i \pm \bar{q}_i) - (q_j \pm \bar{q}_j)\,, \qquad \text{(non-singlets)}$$

$$q_{{\rm ns},i}^{\pm} = \sum_{j=1}^{n_f} q_{{\rm ns},ij}^{\pm}\ , \qquad (i = 1, ..., n_f)$$

$$\delta q_{\rm ns}^{\pm} = \sum_{i\in{\rm up},\ j\in{\rm down}} q_{{\rm ns},ij}^{\pm}$$

- Wilson coefficients (up to the second order in α_s):

$$C^{\mathrm{v}}_{3,\mathrm{ns}} = C^{-}_{3,\mathrm{ns}}$$

$$\begin{aligned} C^{\pm}_{i,\mathrm{ns}} &= \delta(1-x) + a_s\Big[c^{(1),\pm}_{i,\mathrm{ns}} + L_M P^{(0),\pm}_{\mathrm{ns}}\Big] \\ &+ a_s^2\Big[c^{(2),\pm}_{i,\mathrm{ns}} + L_M\Big(P^{(1),\pm}_{\mathrm{ns}} + c^{(1),\pm}_{i,\mathrm{ns}} \otimes (P^{(0),\pm}_{\mathrm{ns}} - \beta_0)\Big) + L_M^2\Big(\frac{1}{2}P^{(0),\pm}_{\mathrm{ns}} \otimes (P^{(0),\pm}_{\mathrm{ns}} - \beta_0)\Big) \\ &+ \beta_0 L_R\Big(c^{(1),\pm}_{i,\mathrm{ns}} + L_M P^{(0),\pm}_{\mathrm{ns}}\Big)\Big]\,, \qquad (i = 1,2,3) \end{aligned}$$

$$\begin{aligned} C_{i,\mathrm{q}} &= \delta(1-x) + a_s\Big[c^{(1)}_{i,\mathrm{q}} + L_M P^{(0)}_{\mathrm{qq}}\Big] \\ &+ a_s^2\Big[c^{(2)}_{i,\mathrm{q}} + L_M\Big(P^{(1)}_{\mathrm{qq}} + c^{(1)}_{i,\mathrm{q}} \otimes (P^{(0)}_{\mathrm{qq}} - \beta_0) + c^{(1)}_{i,\mathrm{g}} \otimes P^{(0)}_{\mathrm{gq}}\Big) \\ &+ L_M^2\Big(\frac{1}{2}P^{(0)}_{\mathrm{qq}} \otimes (P^{(0)}_{\mathrm{qq}} - \beta_0) + \frac{1}{2}P^{(0)}_{\mathrm{qg}} \otimes P^{(0)}_{\mathrm{gq}}\Big) \\ &+ \beta_0 L_R\Big(c^{(1)}_{i,\mathrm{q}} + L_M P^{(0)}_{\mathrm{qq}}\Big)\Big]\,, \qquad (i = 1,2) \end{aligned}$$

$$\begin{aligned} C_{i,\mathrm{g}} &= a_s\Big[c^{(1)}_{i,\mathrm{g}} + L_M P^{(0)}_{\mathrm{qg}}\Big] \\ &+ a_s^2\Big[c^{(2)}_{i,\mathrm{g}} + L_M\Big(P^{(1)}_{\mathrm{qg}} + c^{(1)}_{i,\mathrm{q}} \otimes P^{(0)}_{\mathrm{qg}} + c^{(1)}_{i,\mathrm{g}} \otimes (P^{(0)}_{\mathrm{gg}} - \beta_0)\Big) \\ &+ L_M^2\Big(\frac{1}{2}P^{(0)}_{\mathrm{qq}} \otimes P^{(0)}_{\mathrm{qg}} + \frac{1}{2}P^{(0)}_{\mathrm{qg}} \otimes (P^{(0)}_{\mathrm{gg}} - \beta_0)\Big) \\ &+ \beta_0 L_R\Big(c^{(1)}_{i,\mathrm{g}} + L_M P^{(0)}_{\mathrm{qg}}\Big)\Big]\,, \qquad (i = 1,2) \end{aligned}$$

where $a_s = \alpha_s(\mu_r)/(4\pi)$, $L_M = \ln(Q^2/\mu_f^2)$, $L_R = \ln(\mu_r^2/\mu_f^2)$.

At 1-loop order:

$$P_{\rm ns}^{(0),\pm} = P_{\rm qq}^{(0)}, \qquad\qquad c_{i,\rm ns}^{(1),\pm} = c_{i,\rm q}^{(1)}, \quad (i=1,2,3)$$

At 2-loop order:

$$P_{\rm qq}^{(1)} = P_{\rm ns}^{(1),+} + P_{\rm ps}^{(1)}, \qquad\qquad c_{i,\rm q}^{(2)} = c_{i,\rm ns}^{(2),+} + c_{i,\rm ps}^{(2)}, \quad (i=1,2)$$

DIS coefficient and splitting functions used:

$$c_{i,\rm q}^{(1)},\ c_{3,\rm q}^{(1)},\quad c_{i,\rm g}^{(1)},\quad c_{i,\rm ns}^{(2),\pm},\quad c_{i,\rm ps}^{(2)},\quad c_{i,\rm g}^{(2)} \quad (i=1,2)$$

$$P_{\rm qq}^{(0)},\quad P_{\rm qg}^{(0)},\quad P_{\rm gq}^{(0)},\quad P_{\rm gg}^{(0)},\quad P_{\rm ns}^{(1),\pm},\quad P_{\rm ps}^{(1)},\ P_{\rm qg}^{(1)}$$

3. VBF Higgs pair production

	$\sin(\beta-\alpha)$	$\tan\beta$	m_{h^0} (GeV)	m_{H^0} (GeV)	m_{A^0} (GeV)	$m_{H^\pm}$ (GeV)
B1	0.6	2	126	275	600	600
B2	1	1.5	126	160	380	420

In order to make a precision comparison between the theoretical predictions and experimental measurements, we should assess thoroughly the theoretical uncertainties affecting the central predictions of the total cross sections.

Scale uncertainty:

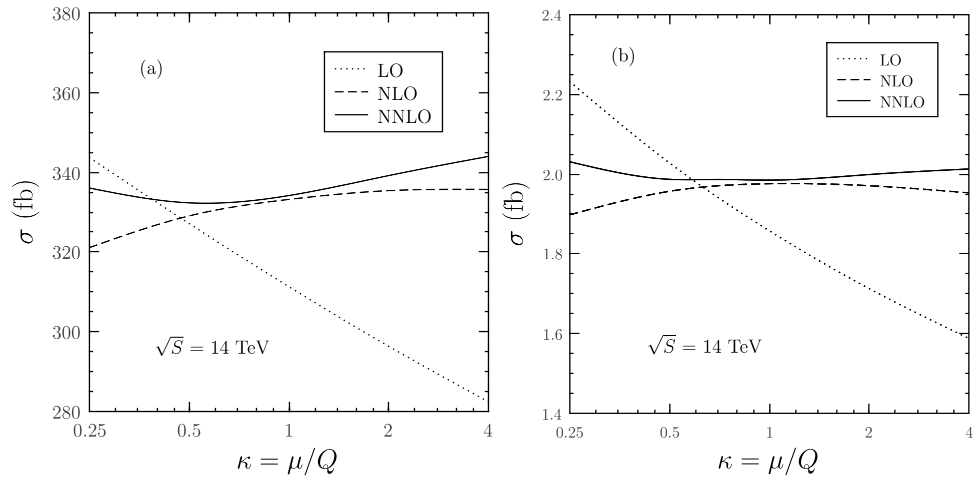


Figure 1: The scale dependence of the LO, QCD NLO and NNLO corrected integrated cross sections for the VBF $h^0h^0 + 2$ *jets* production at the $\sqrt{S} = 14$ TeV LHC. (a) at the benchmark point B1. (b) at the benchmark point B2.

	σ_{LO} (fb)	σ_{NLO} (fb)	σ_{NNLO} (fb)
B1	$311.30^{+32.38\ (+10\%)}_{-28.88\ (-9\%)}$	$333.20^{+2.51\ (+0.8\%)}_{-12.08\ (-3.6\%)}$	$334.18^{+9.82\ (+2.9\%)}_{-1.83\ (-0.5\%)}$
B2	$1.858^{+0.374\ (+20\%)}_{-0.270\ (-15\%)}$	$1.976^{+0.00\ (+0.0\%)}_{-0.078\ (-3.9\%)}$	$1.986^{+0.045\ (+2.3\%)}_{-0.00\ (-0.0\%)}$

PDF+α_s uncertainty:

- For a given parametrization of the PDFs such as MSTW2008, the PDF uncertainty comes from the experimental uncertainties on the fitted data.
- For a fixed value of α_s, MSTW2008 provides a central PDF set S_0 and $2n$ eigenvector PDF sets $S_i^{\pm}$ $(i = 1, ..., n,\ n = 20)$.

(1) The PDF uncertainties on the hadronic cross section are given by

$$\left(\Delta\sigma^{\alpha_s}_{PDF}\right)_+ = \sqrt{\sum_{i=1}^{n}\left\{\max\left[\sigma^{\alpha_s}(S_i^+) - \sigma^{\alpha_s}(S_0),\ \sigma^{\alpha_s}(S_i^-) - \sigma^{\alpha_s}(S_0),\ 0\right]\right\}^2},$$

$$\left(\Delta\sigma^{\alpha_s}_{PDF}\right)_- = \sqrt{\sum_{i=1}^{n}\left\{\max\left[\sigma^{\alpha_s}(S_0) - \sigma^{\alpha_s}(S_i^+),\ \sigma^{\alpha_s}(S_0) - \sigma^{\alpha_s}(S_i^-),\ 0\right]\right\}^2}$$

where $\sigma^{\alpha_s}(S_0)$, $\sigma^{\alpha_s}(S_i^+)$ and $\sigma^{\alpha_s}(S_i^-)$ represent the cross sections obtained by using the PDF sets S_0, S_i^+ and S_i^-, respectively.

(2) Pure α_s uncertainties:

$$
\begin{aligned}
(\Delta\sigma_{\alpha_s})_+ &= \max_{\alpha_s}\left[\sigma^{\alpha_s}(S_0)\right] - \sigma^{\alpha_s^0}(S_0), \\
(\Delta\sigma_{\alpha_s})_- &= \sigma^{\alpha_s^0}(S_0) - \min_{\alpha_s}\left[\sigma^{\alpha_s}(S_0)\right]
\end{aligned}
$$

where max and min run over the five values of α_s: $\alpha_s = \alpha_s^0 \pm 0.5\sigma, \ \pm 1\sigma$.

(3) Combined PDF+α_s uncertainties:

$$
\begin{aligned}
(\Delta\sigma_{PDF+\alpha_s})_+ &= \max_{\alpha_s}\left[\sigma^{\alpha_s}(S_0) + \left(\Delta\sigma^{\alpha_s}_{PDF}\right)_+\right] - \sigma^{\alpha_s^0}(S_0), \\
(\Delta\sigma_{PDF+\alpha_s})_- &= \sigma^{\alpha_s^0}(S_0) - \min_{\alpha_s}\left[\sigma^{\alpha_s}(S_0) - \left(\Delta\sigma^{\alpha_s}_{PDF}\right)_-\right]
\end{aligned}
$$

The dependence of $\left(\Delta\sigma^{\alpha_s}_{PDF}\right)_\pm$ on α_s is negligible,

$$
(\Delta\sigma_{PDF+\alpha_s})_\pm \simeq (\Delta\sigma_{PDF})_\pm + (\Delta\sigma_{\alpha_s})_\pm
$$

$\sqrt{S}$ (TeV)		σ_{LO} (fb)	σ_{NLO} (fb)	σ_{NNLO} (fb)
14	B1	$311.30^{+32.38+4.06}_{-28.88-4.04}\begin{pmatrix}+10.4\%+1.3\% \\ -9.3\%-1.3\%\end{pmatrix}$	$333.20^{+2.51+8.46}_{-12.08-6.57}\begin{pmatrix}+0.8\%+2.5\% \\ -3.6\%-2.0\%\end{pmatrix}$	$334.18^{+9.82+7.36}_{-1.83-5.87}\begin{pmatrix}+2.9\%+2.2\% \\ -0.5\%-1.8\%\end{pmatrix}$
	B2	$1.858^{+0.374+0.028}_{-0.270-0.026}\begin{pmatrix}+20.1\%+1.5\% \\ -14.5\%-1.4\%\end{pmatrix}$	$1.976^{+0+0.052}_{-0.078-0.039}\begin{pmatrix}+0\%+2.6\% \\ -3.9\%-2.0\%\end{pmatrix}$	$1.986^{+0.045+0.048}_{-0-0.035}\begin{pmatrix}+2.3\%+2.4\% \\ -0.0\%-1.8\%\end{pmatrix}$
33	B1	$1404^{+0+15}_{-30-16}\begin{pmatrix}+0\%+1.1\% \\ -2.1\%-1.1\%\end{pmatrix}$	$1500^{+54+35}_{-74-32}\begin{pmatrix}+3.6\%+2.3\% \\ -4.9\%-2.1\%\end{pmatrix}$	$1503^{+73+32}_{-17-28}\begin{pmatrix}+4.9\%+2.1\% \\ -1.1\%-1.9\%\end{pmatrix}$
	B2	$11.234^{+0.878+0.129}_{-0.830-0.149}\begin{pmatrix}+7.8\%+1.1\% \\ -7.4\%-1.3\%\end{pmatrix}$	$12.002^{+0.190+0.297}_{-0.562-0.225}\begin{pmatrix}+1.6\%+2.5\% \\ -4.7\%-1.9\%\end{pmatrix}$	$12.041^{+0.359+0.258}_{-0.060-0.209}\begin{pmatrix}+3.0\%+2.1\% \\ -0.5\%-1.7\%\end{pmatrix}$
100	B1	$7271^{+770+73}_{-1130-81}\begin{pmatrix}+10.6\%+1.0\% \\ -15.5\%-1.1\%\end{pmatrix}$	$7554^{+535+188}_{-580-119}\begin{pmatrix}+7.1\%+2.5\% \\ -7.7\%-1.6\%\end{pmatrix}$	$7578^{+553+150}_{-134-170}\begin{pmatrix}+7.3\%+2.0\% \\ -1.8\%-2.2\%\end{pmatrix}$
	B2	$75.36^{+4.91+2.07}_{-6.34-1.07}\begin{pmatrix}+6.5\%+2.7\% \\ -8.4\%-1.4\%\end{pmatrix}$	$79.82^{+3.92+2.99}_{-5.26-1.95}\begin{pmatrix}+4.9\%+3.7\% \\ -6.6\%-2.4\%\end{pmatrix}$	$80.05^{+3.92+1.58}_{-0.80-1.48}\begin{pmatrix}+4.9\%+2.0\% \\ -1.0\%-1.8\%\end{pmatrix}$

Model parameter dependence:

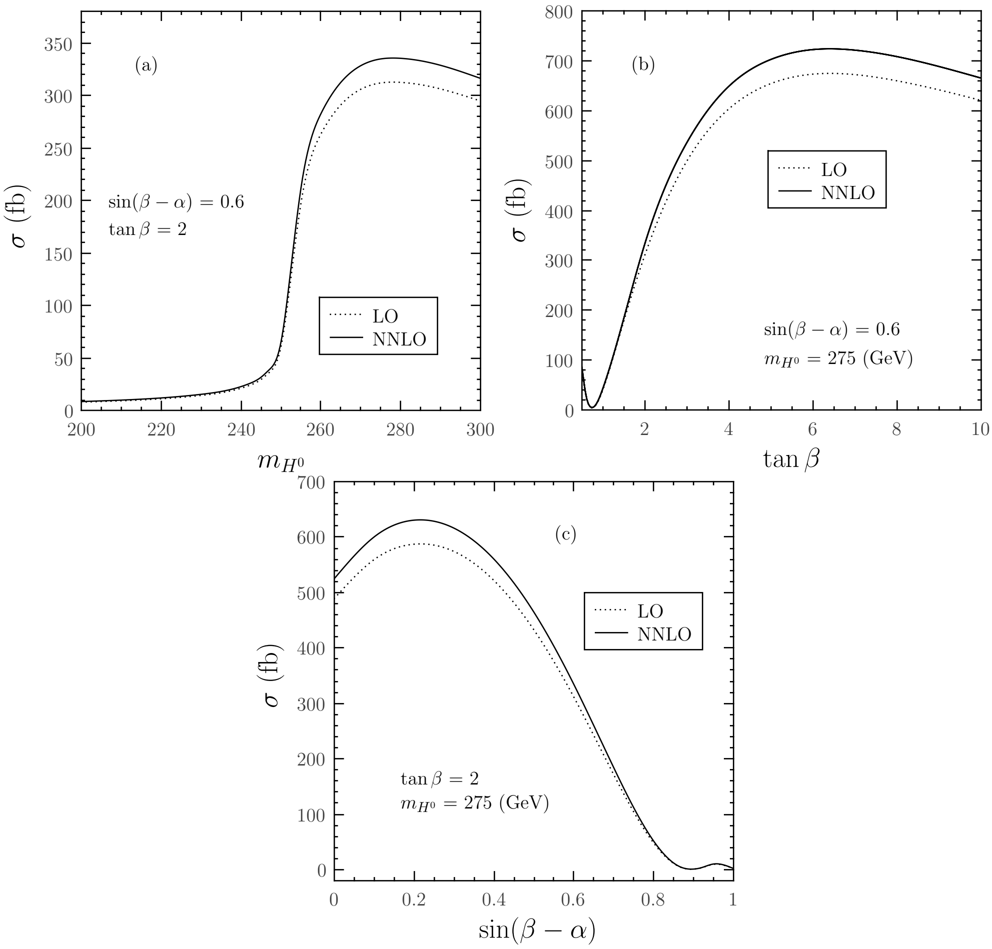


Figure 2: The dependence of the LO and NNLO QCD corrected integrated cross sections for the VBF $h^0h^0 + 2\ jets$ production on the 2HDM(II) parameters at the $\sqrt{S} = 14$ TeV LHC.

Kinematic distributions:

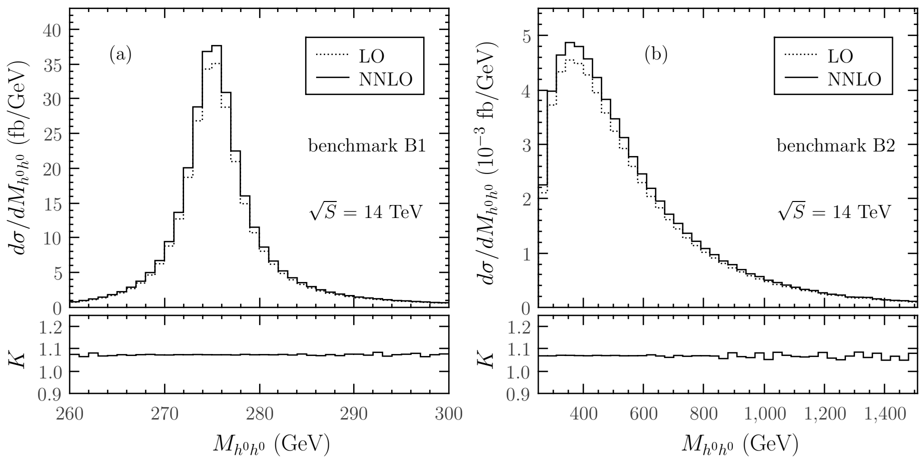


Figure 3: Higgs pair invariant mass distributions.

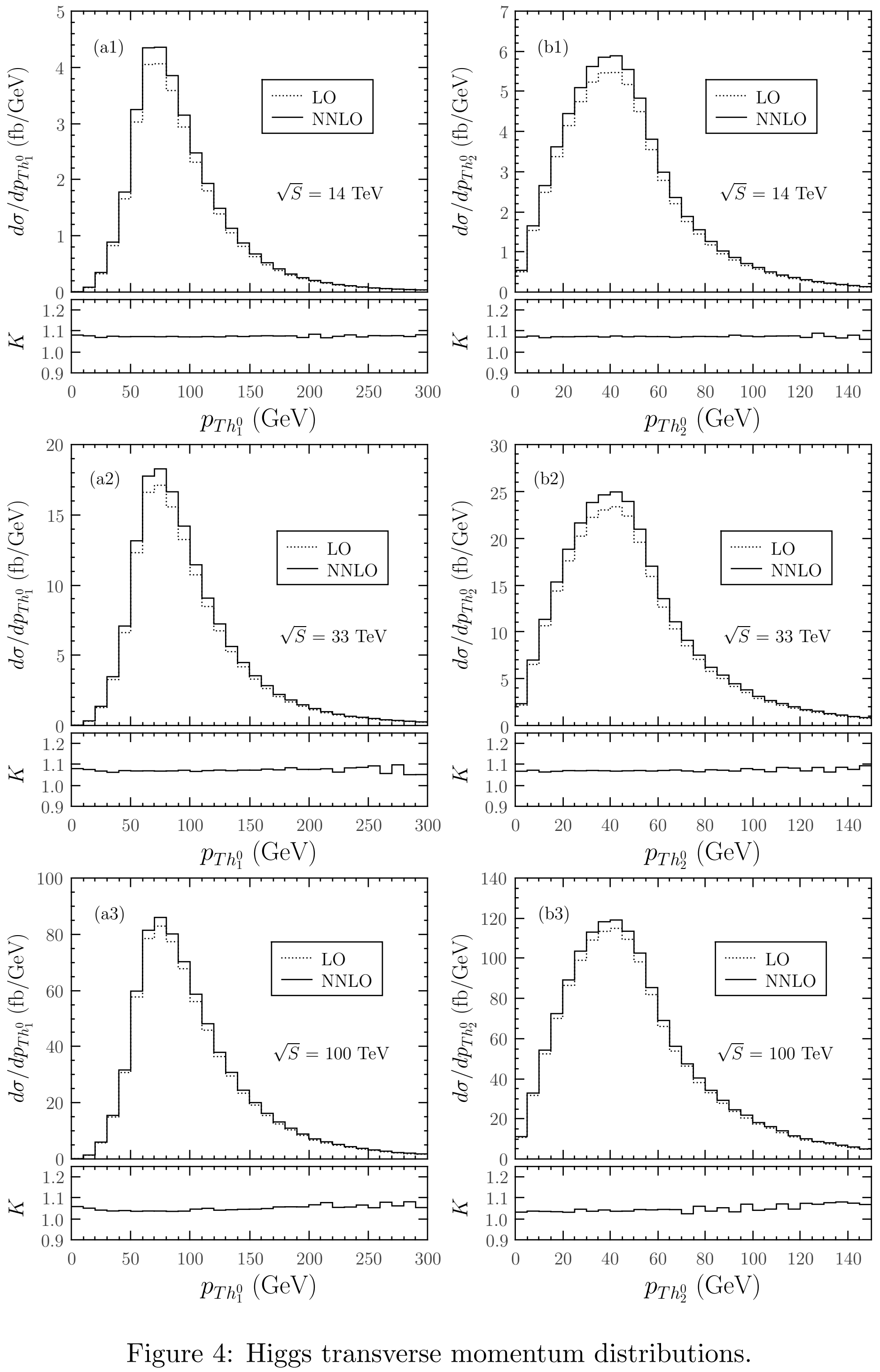


Figure 4: Higgs transverse momentum distributions.

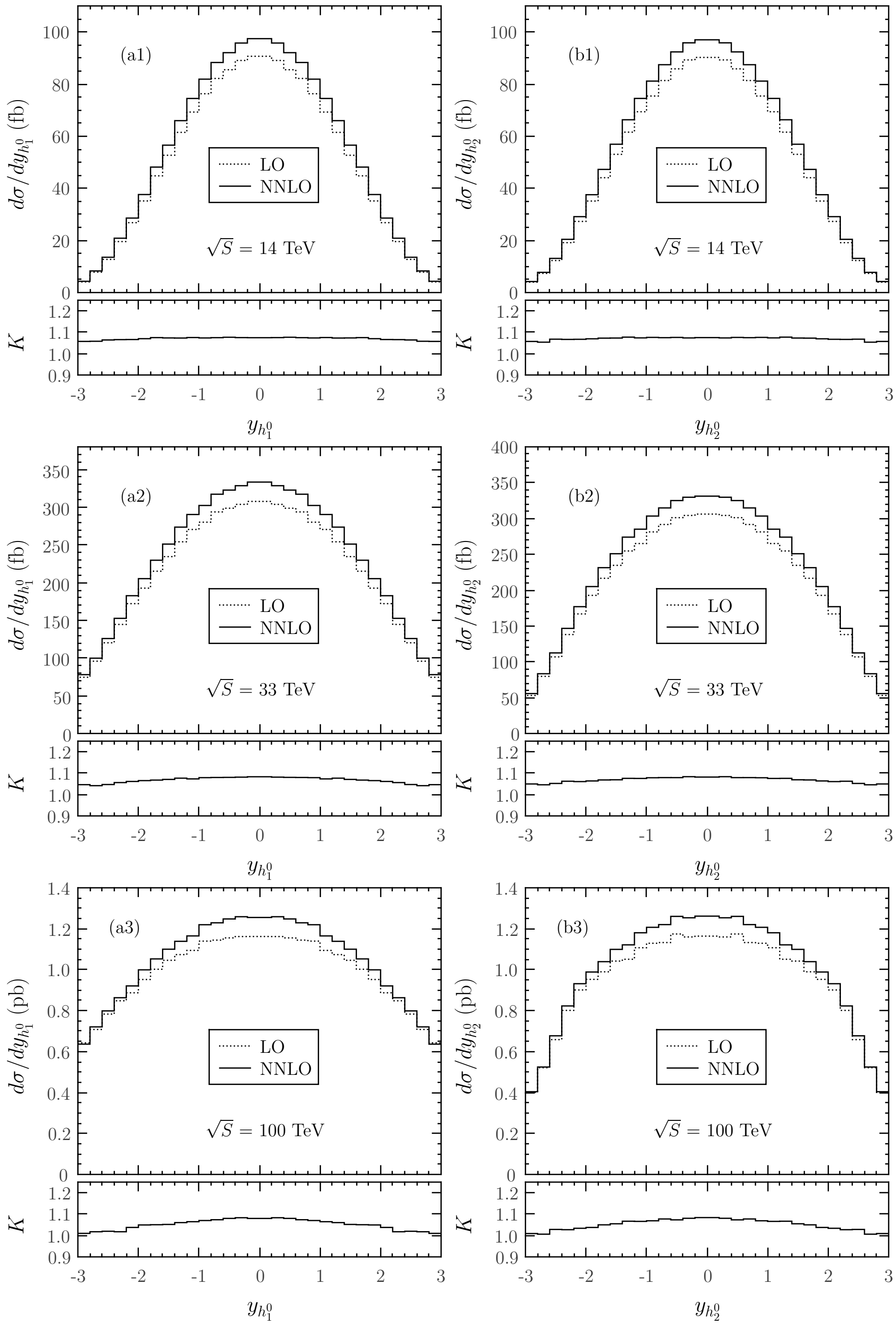


Figure 5: Higgs rapidity distributions.