量子场论 第一讲 符号与约定

$$c = 3\times 10^8\mathrm{m/s},\quad \hbar = 10^{-34}\mathrm{J\cdot s},\quad 1\mathrm{e} = 1.6\times 10^{-19}\mathrm{C}\quad\Rightarrow\quad 1\mathrm{eV} = 1.6\times 10^{-19}\mathrm{J},\quad 1\mathrm{GeV} = 1.6\times 10^{-10}\mathrm{J}$$

$$1\mathrm{GeV/c^2} = 1.6\times 10^{-10}\mathrm{J}/(3\times 10^8\mathrm{m/s})^2 = 2\times 10^{-27}\mathrm{kg},\quad 1\mathrm{fm} = 10^{-15}\mathrm{m},\quad 1\mathrm{b}\equiv(10\mathrm{fm})^2 = 100\mathrm{fm}^2$$

$$1\mathrm{mb} = 10^{-3}\mathrm{b} = 0.1\mathrm{fm}^2,\quad 1\mu\mathrm{b} = 10^{-6}\mathrm{b},\quad 1\mathrm{pb} = 10^{-12}\mathrm{b} = 10^{-10}\mathrm{fm}^2,\quad 1\mathrm{pb}^{-1} = 10^9\mathrm{mb}^{-1}$$

natural units: $\hbar = c = 1 \quad\Rightarrow\quad [\mathrm{length}] = [\mathrm{time}] = [\mathrm{energy}]^{-1} = [\mathrm{mass}]^{-1}$

$$\hbar = 6.6\times 10^{-25}\mathrm{GeV\cdot s}\quad\Rightarrow\quad 1\mathrm{GeV}^{-1} = 6.6\times 10^{-25}\mathrm{s}$$

$$\hbar c = 0.2\mathrm{GeV\cdot fm}\quad\Rightarrow\quad 1\mathrm{GeV}^{-1} = 0.2\mathrm{fm}$$

$$(\hbar c)^2 = 0.4\mathrm{GeV}^2\cdot\mathrm{mb}\quad\Rightarrow\quad 1\mathrm{GeV}^{-2} = 0.4\mathrm{mb},\quad 1\mathrm{mb}^{-1} = 0.4\mathrm{GeV}^2$$

$$x^\mu = (t,\vec{x}),\ \ \partial_\mu = (\partial_t,\nabla),\ \ \eta_{\mu\nu} = \eta^{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)\ \Rightarrow\ x_\mu = \eta_{\mu\nu}x^\nu = (t,-\vec{x}),\ \ \partial^\mu = (\partial_t,-\nabla)$$

$$p^\mu = (E,\vec{p})\ \Rightarrow\ p_\mu = \eta_{\mu\nu}p^\nu = (E,-\vec{p}),\quad p_\mu x^\mu = Et - \vec{p}\cdot\vec{x},\quad p_\mu p^\mu = E^2 - \vec{p}^2 = m^2$$

$$\epsilon^{0123} = 1\ \Rightarrow\ \epsilon^{1230} = -1,\quad \epsilon_{0123} = \eta_{0\mu}\eta_{1\nu}\eta_{2\rho}\eta_{3\sigma}\epsilon^{\mu\nu\rho\sigma} = \eta_{00}\eta_{11}\eta_{22}\eta_{33}\epsilon^{0123} = -1$$

operators: $E = i\partial_t,\ \vec{p} = -i\nabla$, i.e. $p^\mu = i\partial^\mu,\quad i\partial^\mu(e^{-ik\cdot x}) = k^\mu e^{-ik\cdot x}$

$$T_\pm \equiv \frac{\sigma_1}{2}\pm\frac{\sigma_2}{2},\ \ \{\sigma_i,\sigma_j\} = 2\delta_{ij},\ \ [\sigma_i,\sigma_j] = 2i\epsilon_{ijk}\sigma_k\ \Rightarrow\ \sigma_i\sigma_j = \frac{1}{2}\{\sigma_i,\sigma_j\} + \frac{1}{2}[\sigma_i,\sigma_j] = \delta_{ij} + i\epsilon_{ijk}\sigma_k$$

$$\int d^4x e^{\pm ikx} = (2\pi)^4\delta^4(k)\ \Rightarrow\ f(x) = \int\frac{d^4k}{(2\pi)^4}e^{-ikx}f(k),\quad f(k) = \int d^4x e^{ikx}f(x)$$

$$A^\mu = (\phi, \vec{A}), \quad F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \;\Rightarrow\; E_i = F_{0i},\; B_i = -\frac{1}{2}\epsilon_{ijk}F_{jk} \;\text{(i.e. } B_1 = -F_{23},\;\text{etc.)}$$

$$j^\mu = (\rho, \vec{j}), \quad \partial_\rho F^{\rho\mu} = j^\mu \quad\Leftrightarrow\quad \nabla\cdot\vec{E} = \rho, \quad \nabla\times\vec{B} - \partial_t\vec{E} = \vec{j}$$

$$\epsilon^{\mu\nu\rho\sigma}\partial_\nu F_{\rho\sigma} = 0 \quad\Leftrightarrow\quad \nabla\cdot\vec{B} = 0, \quad \nabla\times\vec{E} + \partial_t\vec{B} = 0$$

$$(i\gamma^\mu\partial_\mu - m)\psi = 0, \quad \{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}\mathbf{1} \;\Rightarrow\; (\gamma^0)^2 = \mathbf{1}, \quad (\gamma^i)^2 = -\mathbf{1}$$

$$\text{Let } \gamma^\mu \text{ be unitary} \;\Rightarrow\; (\gamma^0)^\dagger = \gamma^0, \quad (\gamma^i)^\dagger = -\gamma^i \;\Leftrightarrow\; (\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0$$

$$\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 \;\Rightarrow\; (\gamma_5)^2 = \mathbf{1}, \quad (\gamma_5)^\dagger = \gamma_5, \quad \{\gamma_5, \gamma^\mu\} = 0$$

$$\mathrm{tr}(\gamma^\mu\gamma^\nu) = 4\eta^{\mu\nu}, \quad \mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4(\eta^{\mu\nu}\eta^{\rho\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\rho}\eta^{\nu\sigma}), \quad \mathrm{tr}(\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = -4i\epsilon^{\mu\nu\rho\sigma}$$

$$\gamma^\mu\gamma_\mu = d, \quad \gamma^\rho\gamma^\mu\gamma_\rho = -(d-2)\gamma^\mu, \quad \gamma^\rho\gamma^\mu\gamma^\nu\gamma_\rho = 4\eta^{\mu\nu} - (4-d)\gamma^\mu\gamma^\nu$$

$$\gamma^\rho\gamma^\mu\gamma^\nu\gamma^\alpha\gamma_\rho = -2\gamma^\alpha\gamma^\nu\gamma^\mu + (4-d)\gamma^\mu\gamma^\nu\gamma^\alpha$$

$$\text{chiral representation: } \gamma^\mu = \begin{pmatrix} & \sigma^\mu \\ \bar{\sigma}^\mu & \end{pmatrix}, \quad \sigma^\mu \equiv (1, \sigma_i), \quad \bar{\sigma}^\mu \equiv (1, -\sigma_i), \quad \gamma_5 = \begin{pmatrix} -1 & \\ & 1 \end{pmatrix}$$

$$\text{standard representation: } \gamma^0 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} & \sigma_i \\ -\sigma_i & \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$$