Composite Higgs model with Dark Matter

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Based on **arXiv:1805.07619** (accepted by JHEP)

Workshop on Particle Physics Frontiers

21-25 January 2019, Guangzhou

The Standard Model

What we know from the Standard Model (SM):

- Fermionic fields: quarks, leptons $\longrightarrow$ **Matter**,
- Vector fields: photon, $W^{\pm}$, Z, gluons $\rightsquigarrow$ **Force**,
- Scalar fields: Higgs boson $\dashrightarrow$ **origin of mass**.

Not explained by SM:

- Why $m_h \ll \Lambda_{GUT}$? (Hierarchy problem),
- Dark energy, **dark matters**,
- Neutrino masses and oscillation,
- Matter—antimatter asymmetry,
- Strong CP problem,...

New physics are needed!

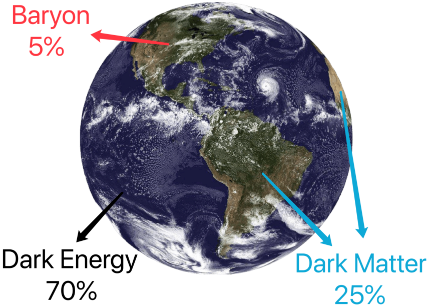
Dark matter

Evidences for dark matter (DM):

- Galaxy rotation curve,
- CMB,
- Bullet cluster,...

Properties of DM:

- Weak interacting,
- Massive, probably cold,
- Stable.



Detections:

- Scattering nucleons (Direct detection),
- Annihilating to SM particles (Indirect detection),
- Producing in colliders, ...

A famous candidate: WIMPs

Fundamental Composite Higgs Model

- $2N_f$ Weyl spinors ψ^i charged under some gauge Group G_{TC}.
- Global symmetry $G_F = \mathrm{SU}(2N_f)$ or $\mathrm{SU}(N_f) \times \mathrm{SU}(N_f)$,
- Non-abelian G_{TC}, asymptotic freedom $\Rightarrow$ ψ^i condense in the IR,

$$\langle \psi^i \psi^j \rangle \sim \Sigma^{ij} \neq 0 \quad \Rightarrow \quad G_F \to H \tag{1}$$

  where H is a subgroup of G_F.
  - ψ^i: real reps. of G_{TC} $\Rightarrow$ $\mathrm{SU}(2N_f) \to \mathrm{SO}(2N_f)$,
  - ψ^i: pseudo-real reps. of G_{TC} $\Rightarrow$ $\mathrm{SU}(2N_f) \to \mathrm{Sp}(2N_f)$.
  - ψ^i: complex reps. of G_{TC} $\Rightarrow$ $\mathrm{SU}(N_f) \times \mathrm{SU}(N_f) \to \mathrm{SU}(N_f)$.
- If Higgs doublet $\subset$ pNGBs, protected by shift symmetry
- SU(4)/Sp(4): minimal model [E. Katz (2005), B. Gripaio (2009), M. Frigerio (2012), G. Cacciapaglia (2014)],
- SU(6)/Sp(6): 2HDM or Singlet-Doublet-Triplet Model.

$\mathrm{Sp}(2N)$ group

- $\mathrm{Sp}(2N) = \mathrm{Sp}(2N, C) \cap \mathrm{SU}(2N)$, $2N \times 2N$ matrices U satisfy

$$UEU^T = E, \qquad E = \begin{pmatrix} & \mathbb{1}_{N\times N} \\ -\mathbb{1}_{N\times N} & \end{pmatrix}, \tag{2}$$

$$or \quad U = e^{i\theta^a \tilde{S}^a}, \qquad \tilde{S}^a E + E(\tilde{S}^a)^T = 0 \tag{3}$$

- Choice of E is not unique,

$$\Sigma_B = \begin{pmatrix} \mathcal{J} & & \\ & \pm\mathcal{J} & \\ & & \ddots \end{pmatrix}, \quad \mathcal{J} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \begin{matrix} \Sigma_B = OEO^T, \quad S^a = O\tilde{S}^a O^{-1}, \\ S^a \Sigma_B + \Sigma_B (S^a)^T = 0 \end{matrix} \tag{4}$$

- Degree of freedom: $\mathrm{SU}(2N) \sim 4N^2 - 1$, $\mathrm{Sp}(2N) \sim 2N^2 + N$, $\mathrm{SU}(2N)/\mathrm{Sp}(2N) \sim (N-1)(2N+1)$.

The minimal model, SU(4)/Sp(4)

4 Weyl spinors, fund. reps of $\mathrm{SU}(2)_{TC}$, $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ reps.

$$Q_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix} \sim (2,0), \quad \tilde{U}_L \sim (1,-1/2), \quad \tilde{D}_L \sim (1,1/2) \tag{5}$$

$(\tilde{U}_L, \tilde{D}_L)$ form a doublet of $\mathrm{SU}(2)_R$, the T_R^3 generator is gauged to be the $U(1)_Y$. The condensation induces a vacuum as a combination of

$$\Sigma_0 = c_\theta \Sigma_B + s_\theta \Sigma_H, \quad \Sigma_B = \begin{pmatrix} i\sigma^2 & 0 \\ 0 & -i\sigma^2 \end{pmatrix}, \quad \Sigma_H = \begin{pmatrix} 0 & 1_{2\times 2} \\ -1_{2\times 2} & 0 \end{pmatrix}, \tag{6}$$

Σ_B respects the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ generators

$$T_L^i = \frac{1}{2}\begin{pmatrix} \sigma^i & 0 \\ 0 & 0 \end{pmatrix}, \qquad T_R^3 = \frac{1}{2}\begin{pmatrix} 0 & 0 \\ 0 & -\sigma^3 \end{pmatrix} \tag{7}$$

Before the EWSB, Σ_B would be chosen by the condensation.

NGBs and chiral Lagrangian

In the IR, 5 relevant NGBs are spanned by the broken generators

$$\Sigma = e^{i\frac{\phi^a}{f}X^a}\Sigma_B \sim \Sigma_B + \frac{i\phi^a}{f}X^a\Sigma_B + O((\phi^a)^2) \quad (8)$$

$$\sim \Sigma_B + \frac{1}{2\sqrt{2}f}\begin{pmatrix}\eta\sigma^2 & \mathcal{H} \\ -\mathcal{H}^T & \eta\sigma^2\end{pmatrix} + \ldots \quad (9)$$

$$\mathcal{H} = (i\sigma^2 H^*, H) \sim \text{Higgs bidoublet}$$

The chiral Lagrangian is

$$\mathcal{L} = f^2 Tr((D_\mu\Sigma)^\dagger D^\mu\Sigma) + V(\Sigma) + \ldots \quad (10)$$

$$V(\Sigma) = V_{gauge} + V_{top} + V_m. \quad (11)$$

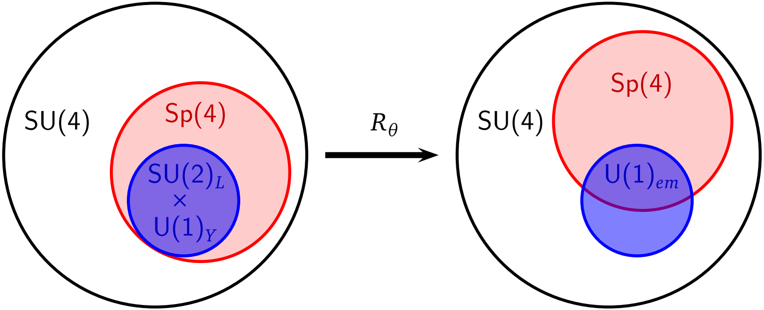
The true vacuum:

$$\frac{\partial V}{\partial\theta} = 0 \Rightarrow s_\theta \neq 0 \text{ (EWSB)} \quad (12)$$

EWSB driven by vacuum misalignment

The potential $V(\Sigma)$ leads to a vacuum misaligned with the EW generators. Parametrize the misalignment with a rotation generated by the "Higgs vev"

$$\Sigma_0 = R_{\theta/2}\Sigma_B R^T_{\theta/2} = \cos\theta\,\Sigma_B + \sin\theta\,\Sigma_H, \quad R_{\theta/2} = e^{i\sqrt{2}\theta X^4} \tag{13}$$



Lagrangian of h, η

$$\mathcal{L} = \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}(\partial_\mu \eta)^2 + \frac{1}{48f^2}[-(h\partial_\mu \eta - \eta \partial_\mu h)^2] + O(f^{-3})$$
$$+ (2g^2 W_\mu^+ W^{-,\mu} + (g^2 + g'^2) Z_\mu Z^\mu)[f^2 s_\theta^2 + \frac{s_{2\theta} f}{2\sqrt{2}} h(1 - \frac{1}{12f^2}(h^2+\eta^2))$$
$$+ \frac{1}{8}(c_{2\theta} h^2 - s_\theta^2 \eta^2)(1 - \frac{1}{24f^2}(h^2+\eta^2)) + O(f^{-3})] \quad (14)$$

$v = 2\sqrt{2} f \sin\theta \approx 246$ GeV.

$$g_{hWW} = g_{hWW}^{SM} c_\theta, \quad g_{hZZ} = g_{hZZ}^{SM} c_\theta \quad (15)$$

- h is a candidate of Higgs boson if θ is small.
- The EW singlet η is a pseudo-scalar field beyond the standard model.
- From (14), it seems that η is protected by Z_2...
  NOT TRUE. It is broken by WZW term [A. Arbey, et al. (2015)].
- Reproducing SM, but no DM candidate.

SU(6) → Sp(6) composite model

- 6 left-handed Weyl spinors ψ, fundamental reps of G_{TC} =SU(2).
- In the IR, $\langle\psi^i\psi^j\rangle \sim \Sigma^{ij}$ antisymmetric, SU(6) →Sp(6).
- NGBs: d.o.f $= 35 - 21 = 14$, decomposition: $(SU(2)_1, SU(2)_2, SU(2)_3)$

$$14_{\mathrm{Sp}(6)} \to (2,2,1) \oplus (2,1,2) \oplus (1,2,2) \oplus (1,1,1) \oplus (1,1,1) \tag{16}$$

Case		$SU(2)_L$	$U(1)_Y$	$SU(2)_L$	Y	Higgs
A	ψ_1	**2**	0	SU$(2)_1$	$T_2^3 + T_3^3$	$(2,2,1) + (2,1,2)$
	ψ_2	**1**	$\pm 1/2$			
	ψ_3	**1**	$\pm 1/2$			
B	ψ_1	**2**	0	SU$(2)_1$ + SU$(2)_3$	T_2^3	$(2,2,1) + (1,2,2)$
	ψ_2	**1**	$\pm 1/2$			
	ψ_3	**2**	0			

[C. Cai, G.Cacciapaglia, H.H. Zhang, (2018)]

The Model A

- Before EW symmetry breaking, $\langle \phi^4 \rangle = \langle \phi^8 \rangle = 0$,

$$\langle \psi^i \psi^j \rangle \sim \Sigma^{ij} = \Sigma_0 = \begin{pmatrix} i\sigma_2 & 0 & 0 \\ 0 & -i\sigma_2 & 0 \\ 0 & 0 & -i\sigma_2 \end{pmatrix} \tag{17}$$

$$i\Pi(\phi') \cdot \Sigma_0 = \frac{1}{2} \begin{pmatrix} S_1 & H_1 & H_2 \\ -H_1^T & S_2 & G \\ -H_2^T & -G^T & S_3 \end{pmatrix} \tag{18}$$

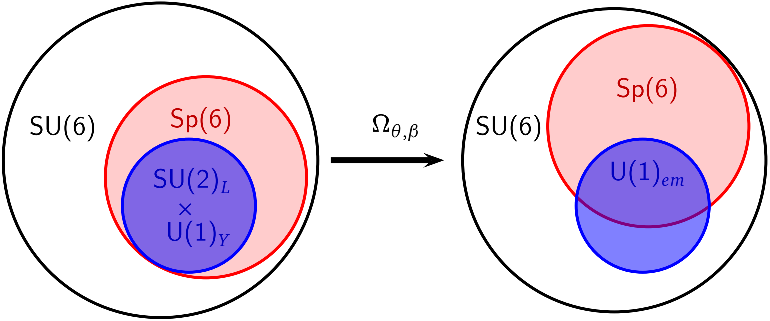
- $H_1 \sim (2,2,1)$, $H_2 \sim (2,1,2)$ —— 2HDM
- $G \sim (1,2,2)$: neutral and charged singlets
- $S_{1,2,3}$: $(1,1,1) \oplus (1,1,1)$ singlets pesudo-scalars

Vacuum misalignment and EWSB

- EW breaking, $\langle\phi^4\rangle = v_1$, $\langle\phi^8\rangle = v_2$, $\tan\beta = v_2/v_1$, $\theta = \sqrt{v_1^2+v_2^2}/2\sqrt{2}f$

$$\Sigma = \Omega_{\theta,\beta}\Sigma_0\Omega^T_{\theta,\beta}, \quad U(\phi)_{\theta,\beta} = \Omega_{\theta,\beta}U_0(\phi)\Omega^\dagger_{\theta,\beta}, \quad \Omega_{\theta,\beta} = R_\beta\Omega_\theta R^\dagger_\beta \tag{19}$$

$$R_\beta = \begin{pmatrix} \mathbb{I}_2 & 0 & 0 \\ 0 & \cos\beta\ \mathbb{1}_2 & -\sin\beta\ \mathbb{1}_2 \\ 0 & \sin\beta\ \mathbb{1}_2 & \cos\beta\ \mathbb{1}_2 \end{pmatrix}, \quad \Omega_\theta = \begin{pmatrix} \cos\frac{\theta}{2}\mathbb{1}_2 & \sin\frac{\theta}{2}\ i\sigma^2 & 0 \\ \sin\frac{\theta}{2}\ i\sigma^2 & \cos\frac{\theta}{2}\ \mathbb{1}_2 & 0 \\ 0 & 0 & \mathbb{I}_2 \end{pmatrix} \tag{20}$$



Composite inert 2HDM

- A special vacuum: all fermions couple to the same $\mathrm{SU}(2)_{R1}$

$$\frac{\partial V}{\partial \beta} = 0 \ + \ \text{tadpoles vanish} \Rightarrow \quad Y_{f2} = 0, \quad \beta = 0, \tag{21}$$

$$\frac{\partial V}{\partial \theta} = 0 \quad \Rightarrow \quad \cos\theta = \frac{16BM/f^4 + 8B\delta m_R/f^4}{2C_t(|y_{t1}|^2 + |y_{b1}|^2) - C_g(3g^2 + g'^2)} \tag{22}$$

- Higgs mass and couplings:

$$m_{h_1}^2 = \frac{C_t}{4} m_t^2 - \frac{C_g}{16}(2m_W^2 + m_Z^2) \sim (125\ \text{GeV})^2 \Rightarrow C_t \sim 2 \tag{23}$$

$$g_{hXX} = g_{hXX}^{\text{SM}} c_\theta, \tag{24}$$

- $\mathrm{SU}(2)_{R2}$ is only broken by gauging $T_{R2}^3 \sim Y$, to a remnant $\mathrm{U}(1)_{DM}$.

$$Q_{DM} = 1 \ \text{fields}: \quad (H^+, H^0) \in (2,1,2), \quad \eta^+, \ \eta^0 \in (1,2,2) \tag{25}$$

- A mixture of H^0 and η^0 can be DM candidate.

Constraints from direct detection

The current DM direct detection experiments set a stringent bound:
$\sigma_{\chi N} \sim 10^{-45}\ \mathrm{cm}^2$ for $m_\chi \sim 1$ TeV (XENON1T,PANDAX II).
$\chi\chi Z$ coupling leads to a χN scattering cross-section as

$$\sigma_{Z,\chi N} \approx \frac{(1-c_\theta)^2 g^4 m_N^4}{16\pi c_W^2 m_Z^2}\frac{1}{2}\left(\frac{1}{4}\right)^2 \sim 5\cdot 10^{-40}(1-c_\theta)^2 \mathrm{cm}^2 \tag{26}$$

which implies a strong constraint on the misalignment angle $s_\theta \lesssim 0.01$.
Any way out?

- Heavier DM and larger fine-tuning (θ smaller);
- Try SU(6)/Sp(6) model B with two $\mathrm{SU}(2)_L$ and one $\mathrm{SU}(2)_R$ (In progress). [C. C, H-H. Z., G. C., M. R. J., M. T. F.].

The Model B

SU(6)/Sp(6) coset

$$\Sigma \sim \Sigma_B + \frac{1}{2f}\begin{pmatrix} A^1 & \mathcal{H}^1 & T \\ -(\mathcal{H}^1)^T & A^2 & -(\mathcal{H}^2)^T \\ -T^T & \mathcal{H}^2 & A^3 \end{pmatrix} + \dots, \tag{27}$$

$$A^1 = \frac{3\sqrt{2}\phi^1 + \sqrt{6}\phi^2}{6}\sigma^2, \quad A^2 = \frac{3\sqrt{2}\phi^1 - \sqrt{6}\phi^2}{6}\sigma^2, \quad A^3 = \frac{2\sqrt{6}\phi^2}{6}\sigma^2,$$

$$\mathcal{H}^{1,2} = \begin{pmatrix} (H^0_{1,2})^* & H^+_{1,2} \\ -H^-_{1,2} & H^0_{1,2} \end{pmatrix}, \quad T = \left[\begin{pmatrix} \frac{\Delta^0}{\sqrt{2}} & \Delta^+ \\ \Delta^- & -\frac{\Delta^0}{\sqrt{2}} \end{pmatrix} - \frac{i\eta^0}{\sqrt{2}} 1_{2\times 2}\right] i\sigma^2$$

$$H^0_2 = \frac{-h_2 + i\chi_2}{\sqrt{2}}$$

A Z_2 parity is preserved under

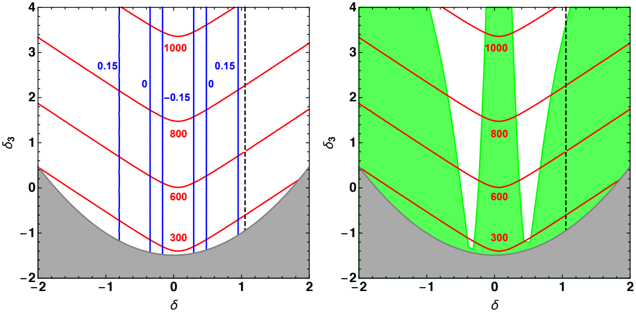
$$H^2 \to -H^2, \quad T \to -T \tag{28}$$

Gauge interaction splits the CP-odd and CP-even sector ⇒ No $\chi\chi Z$ coupling.

Constraints from direct detection

There are Higgs mediating DM-Nucleon scatterings.

$$g_{h\mathcal{X}_1\mathcal{X}_1} = \frac{2t_\alpha^2}{1+t_\alpha^2} g_{h\chi_2\chi_2} + \frac{2}{1+t_\alpha^2} g_{h\eta^0\eta^0} + \frac{2t_\alpha}{1+t_\alpha^2} g_{h\chi_2\eta^0} \tag{29}$$



$$s_\theta = 0.2,\ C_g/C_t = 1/3,\ M_{1,2} = M(1\pm\delta),\ M_3 = M(1+\delta_3)$$

Summary

1. The fundamental composite Higgs model can partly solve the Hierarchy problem.
2. The minimal model, $SU(4)/Sp(4)$, has NO DM candidate.
3. There are two types of $SU(6)/Sp(6)$ models
   - 1 SU(2)$_L$ 2 SU(2)$_R$: 2HDM with U(1)$_{DM}$ symmetry stabilizing the DM candidate. Stringent bound from current direct detection experiment. Need more fine-tuning.
   - 2 SU(2)$_L$ 1 SU(2)$_R$: Singlet-Doublet-Triplet with Z_2 symmetry. DM candidate has loose direct detection bound.
4. For $SU(6)/Sp(6)$ models, many phenomenologies can be studied in the future.

Summary

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4. For $SU(6)/Sp(6)$ models, many phenomenologies can be studied in the future.

Thanks for your attention!

Unbroken generators of Sp(6) for 2HDM model

$$S^1=\frac{1}{2}\begin{pmatrix}\sigma_1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\quad S^2=\frac{1}{2}\begin{pmatrix}\sigma_2 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\quad S^3=\frac{1}{2}\begin{pmatrix}\sigma_3 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}, \tag{30}$$

$$S^4=\frac{1}{2}\begin{pmatrix}0 & 0 & 0\\ 0 & -\sigma_1^T & 0\\ 0 & 0 & 0\end{pmatrix},\quad S^5=\frac{1}{2}\begin{pmatrix}0 & 0 & 0\\ 0 & -\sigma_2^T & 0\\ 0 & 0 & 0\end{pmatrix},\quad S^6=\frac{1}{2}\begin{pmatrix}0 & 0 & 0\\ 0 & -\sigma_3^T & 0\\ 0 & 0 & 0\end{pmatrix}, \tag{31}$$

$$S^7=\frac{1}{2}\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\sigma_1^T\end{pmatrix},\quad S^8=\frac{1}{2}\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\sigma_2^T\end{pmatrix},\quad S^9=\frac{1}{2}\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\sigma_3^T\end{pmatrix}, \tag{32}$$

$$S^{10}=\frac{1}{2\sqrt{2}}\begin{pmatrix}0 & i\sigma_1 & 0\\ -i\sigma_1 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\quad S^{11}=\frac{1}{2\sqrt{2}}\begin{pmatrix}0 & i\sigma_2 & 0\\ -i\sigma_2 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\quad S^{12}=\frac{1}{2\sqrt{2}}\begin{pmatrix}0 & i\sigma_3 & 0\\ -i\sigma_3 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\quad S^{13}=\frac{1}{2\sqrt{2}}\begin{pmatrix}0 & \mathbb{I}_2 & 0\\ \mathbb{I}_2 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}, \tag{33}$$

$$S^{14}=\frac{1}{2\sqrt{2}}\begin{pmatrix}0 & 0 & i\sigma_1\\ 0 & 0 & 0\\ -i\sigma_1 & 0 & 0\end{pmatrix},\quad S^{15}=\frac{1}{2\sqrt{2}}\begin{pmatrix}0 & 0 & i\sigma_2\\ 0 & 0 & 0\\ -i\sigma_2 & 0 & 0\end{pmatrix},\quad S^{16}=\frac{1}{2\sqrt{2}}\begin{pmatrix}0 & 0 & i\sigma_3\\ 0 & 0 & 0\\ -i\sigma_3 & 0 & 0\end{pmatrix},\quad S^{17}=\frac{1}{2\sqrt{2}}\begin{pmatrix}0 & 0 & \mathbb{I}_2\\ 0 & 0 & 0\\ \mathbb{I}_2 & 0 & 0\end{pmatrix}, \tag{34}$$

$$S^{18}=\frac{1}{2\sqrt{2}}\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & \sigma_1\\ 0 & \sigma_1 & 0\end{pmatrix},\quad S^{19}=\frac{1}{2\sqrt{2}}\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & \sigma_2\\ 0 & \sigma_2 & 0\end{pmatrix},\quad S^{20}=\frac{1}{2\sqrt{2}}\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & \sigma_3\\ 0 & \sigma_3 & 0\end{pmatrix},\quad S^{21}=\frac{1}{2\sqrt{2}}\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & i\mathbb{I}_2\\ 0 & -i\mathbb{I}_2 & 0\end{pmatrix}, \tag{35}$$

Broken generators

$$X^1 = \frac{1}{2\sqrt{2}}\begin{pmatrix} \mathbb{I}_2 & 0 & 0 \\ 0 & -\mathbb{I}_2 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad X^2 = \frac{1}{2\sqrt{6}}\begin{pmatrix} \mathbb{I}_2 & 0 & 0 \\ 0 & \mathbb{I}_2 & 0 \\ 0 & 0 & -2\mathbb{I}_2 \end{pmatrix}, \tag{36}$$

$$X^3 = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & \sigma_1 & 0 \\ \sigma_1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad X^4 = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & \sigma_2 & 0 \\ \sigma_2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad X^5 = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & \sigma_3 & 0 \\ \sigma_3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad X^6 = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & i\mathbb{I}_2 & 0 \\ -i\mathbb{I}_2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{37}$$

$$X^7 = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & 0 & \sigma_1 \\ 0 & 0 & 0 \\ \sigma_1 & 0 & 0 \end{pmatrix}, \quad X^8 = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & 0 & \sigma_2 \\ 0 & 0 & 0 \\ \sigma_2 & 0 & 0 \end{pmatrix}, \quad X^9 = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & 0 & \sigma_3 \\ 0 & 0 & 0 \\ \sigma_3 & 0 & 0 \end{pmatrix}, \quad X^{10} = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & 0 & i\mathbb{I}_2 \\ 0 & 0 & 0 \\ -i\mathbb{I}_2 & 0 & 0 \end{pmatrix}, \tag{38}$$

$$X^{11} = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & i\sigma_1 \\ 0 & -i\sigma_1 & 0 \end{pmatrix}, \quad X^{12} = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & i\sigma_2 \\ 0 & -i\sigma_2 & 0 \end{pmatrix}, \quad X^{13} = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & i\sigma_3 \\ 0 & -i\sigma_3 & 0 \end{pmatrix}, \quad X^{14} = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \mathbb{I}_2 \\ 0 & \mathbb{I}_2 & 0 \end{pmatrix}. \tag{39}$$

WZW terms for SU(6)/Sp(6) 2HDM model

The Wess-Zumino-Witten topological term [Wess (1971), Witten (1983)] reads:

$$\mathcal{L}_{WZW} = \frac{d_{\rm FCD}\, g_{V_1 V_2}}{16\sqrt{2}\pi^2 f} \left(c_\theta\ \eta_1 + \frac{1}{\sqrt{3}c_\theta}\ \eta_2 \right) \epsilon_{\mu\nu\rho\sigma} V_1^{\mu\nu} V_2^{\rho\sigma}\,, \tag{40}$$

where $d_{\rm FCD}$ is the dimension of the FCD representation of the underlying fermions ($d_{\rm FCD} = 2$ in the minimal $SU(2)_{\rm TC}$ model), and

$$g_{WW} = g^2, \quad g_{ZZ} = (g^2 - g'^2), \quad g_{Z\gamma} = g g'\,. \tag{41}$$

The couplings above also shows that η_1 and η_2 are pseudo-scalars under CP.

Projectors for the top mass

$$P_1^1 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad P_1^2 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{42}$$

$$P_2^1 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad P_2^2 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{43}$$

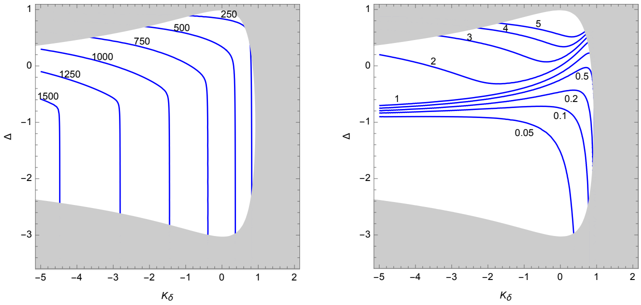
Similar for the bottom Yukawas.

Masses spectrum

- The neutral/charged components mass matrices in basis $(H^0,\eta^0)/(H^\pm,\eta^\pm)$

$$\mathcal{M}^2_{\text{neut.}}=\frac{C_tY_{t1}^2f^2}{8}\begin{pmatrix}(1+c_\theta)-\frac{8K_\delta}{Y_{t1}^2} & s_\theta\\ s_\theta & (1+c_\theta-2\Delta c_\theta)-\frac{4(1-\Delta)K_\delta}{Y_{t1}^2}-\frac{C_g(3g^2+g'^2)}{C_tY_{t1}^2}(1-\Delta)c_\theta\end{pmatrix}$$

$$\mathcal{M}^2_{\text{charg.}}=\mathcal{M}^2_{\text{neut.}}+\frac{C_gg'^2f^2}{4}\begin{pmatrix}(1-c_\theta) & 0\\ 0 & (1+c_\theta)\end{pmatrix}$$

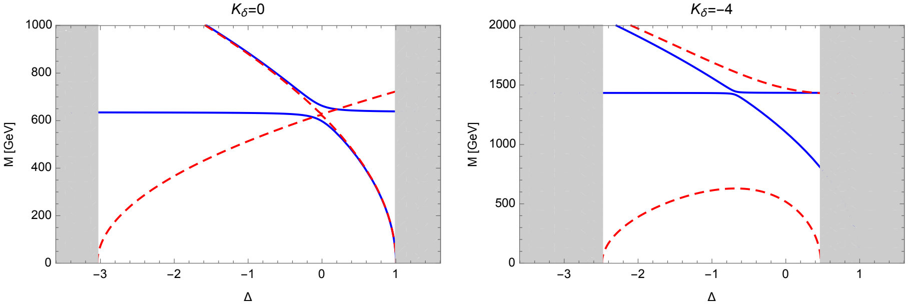


Fixing
$C_g=\frac{1}{3}C_t,\ \theta=0.2$
$(2\sqrt{2}f=1.2$ TeV$)$

Masses spectrum of the pseudo-scalars

- The pseudo-scalars' mass matrix

$$\mathcal{M}_\eta^2 = \frac{m_h^2}{s_\theta^2}\begin{pmatrix} 1 & \frac{1}{\sqrt{3}}\Delta c_\theta \\ \frac{1}{\sqrt{3}}\Delta c_\theta & \frac{1}{3}(2(1-\Delta)+c_\theta)c_\theta \end{pmatrix} - C_t f^2 K_\delta \begin{pmatrix} 0 & \frac{1+\Delta}{2\sqrt{3}} \\ \frac{1+\Delta}{2\sqrt{3}} & \frac{3-\Delta}{3} \end{pmatrix} \qquad (44)$$



The most general vacuum alignment

- The vacuum $\Sigma = \Omega_{\theta,\beta,\gamma} \cdot \Sigma_0 \cdot \Omega^{\dagger}_{\theta,\beta,\gamma}$, $\Omega_{\theta,\beta,\gamma} = R_\beta \cdot R_\gamma \cdot \Omega_\theta \cdot R^{\dagger}_\gamma \cdot R^{\dagger}_\beta$

$$R_\beta = e^{i2\sqrt{2}\beta\ S^{21}}, \quad \Omega_\theta = e^{i\sqrt{2}\theta\ X^4}, \quad R_\gamma = e^{-i2\sqrt{2}\gamma\ S^{14}} = \begin{pmatrix} \cos\gamma\ \mathbb{I}_2 & 0 & \sin\gamma\ \sigma^1 \\ 0 & \mathbb{I}_2 & 0 \\ -\sin\gamma\ \sigma^1 & 0 & \cos\gamma\ \mathbb{I}_2 \end{pmatrix} \tag{45}$$

- EW vev: $v_{\mathrm{SM}} = 2\sqrt{2}f\sin\tau$, $\sin\frac{\tau}{2} \equiv \cos\gamma\sin\frac{\theta}{2}$,
- Higgs mixing:

$$h_1' = \frac{1}{\cos\frac{\tau}{2}}\left(\cos\gamma\ \cos\frac{\theta}{2}\ h_1 - \sin\gamma\ \varphi^0\right), \quad \varphi_0' = \frac{1}{\cos\frac{\tau}{2}}\left(\sin\gamma\ h_1 + \cos\gamma\ \cos\frac{\theta}{2}\ \varphi_0\right) \tag{46}$$

$$\frac{g_{h_1'WW}}{g^{\mathrm{SM}}_{hWW}} = \frac{g_{h_1'ZZ}}{g^{\mathrm{SM}}_{hZZ}} = \cos\tau \tag{47}$$

- Top mass and Yukawa coupling,

$$m_t = 2\cos\frac{\gamma}{2}\ \sin\frac{\theta}{2}\left(Y_{t2}\sin\frac{\gamma}{2} + Y_{t1}\cos\frac{\gamma}{2}\ \cos\frac{\theta}{2}\right) = f\sin(\tau)Y_{\mathrm{top}} \tag{48}$$

$$Y_{\mathrm{top}} = \frac{1}{\cos\frac{\tau}{2}}\left(Y_{t2}\sin\gamma\ \sin\frac{\theta}{2} + Y_{t1}\cos\frac{\theta}{2}\right) \tag{49}$$

Three origins of the pNGBs potential

- Gauge loops

$$V_g = -C_g f^4 \left\{ \frac{g'^2}{2} + \frac{3g^2 + g'^2}{2} c_\theta^2 - \frac{h_1}{2\sqrt{2}} \frac{3g^2+g'^2}{2} s_{2\theta} + \dots \right\} \tag{50}$$

- Fermions' loops

$$V_{\text{Yuk}} = -C_t f^4 \Bigg\{ \left(|Y_{t1}|^2 + |Y_{b1}|^2\right) s_\theta^2 + \frac{h_1}{2\sqrt{2}f}\left(|Y_{t1}|^2 + |Y_{b1}|^2\right) s_{2\theta} + \frac{h_2}{\sqrt{2}f}\left(\Re\ Y_{t1}Y_{t2}^* + \Re\ Y_{b1}Y_{b2}^*\right) c_{\frac{\theta}{2}} s_\theta + \frac{\varphi_0}{\sqrt{2}f}\left(\Re\ Y_{t1}Y_{t2}^* - \Re\ Y_{b1}Y_{b2}^*\right) s_{\frac{\theta}{2}} s_\theta + \frac{A_0}{\sqrt{2}f}\left(\Im\ Y_{t1}Y_{t2}^* - \Im\ Y_{b1}Y_{b2}^*\right) c_{\frac{\theta}{2}} s_\theta + \frac{\eta_3}{\sqrt{2}f}\left(\Im\ Y_{t1}Y_{t2}^* + \Im\ Y_{b1}Y_{b2}^*\right) s_{\frac{\theta}{2}} s_\theta \Bigg\} \tag{51}$$

- Explicitly breaking mass term, $\left(M \equiv \frac{m_L+m_R}{2},\ \delta m_R \equiv \frac{m_{R1}-m_{R2}}{2},\ \Delta \equiv \frac{m_L-m_R}{m_L+m_R}\right)$

$$V_{\text{m}} = -8B \Bigg\{ M\ (1 - \Delta + 2c_\theta) - \delta m_R\ c_{2\beta}\ (1 - c_\theta) - \frac{h_1}{2\sqrt{2}f}\left(2M + \delta m_R c_{2\beta}\right) s_\theta + \frac{h_2}{\sqrt{2}f} \delta m_R s_{2\beta} s_{\frac{\theta}{2}} + \dots \Bigg\} \tag{52}$$

Mass matrices of 2L1R model

$$\mathcal{L} \supset -\frac{1}{2}(h_2, \Delta^0) M_{DT,0}^2 \begin{pmatrix} h_2 \\ \Delta^0 \end{pmatrix} - \frac{1}{2}(\chi_2, \eta^0) M_{DS,0}^2 \begin{pmatrix} \chi_2 \\ \eta^0 \end{pmatrix} - (H_2^+, \Delta^+) M_{DT,\pm}^2 \begin{pmatrix} H_2^- \\ \Delta^- \end{pmatrix}$$

$$M_{DT,0}^2 = M_{DS,0}^2 + C_g g^2 f^2 \begin{pmatrix} -s_{\theta/2}^2 & 0 \\ 0 & c_{\theta/2}^2 \end{pmatrix}, \tag{53}$$

$$M_{DT,\pm}^2 = M_{DS,0}^2 + C_g g^2 f^2 \begin{pmatrix} 0 & 0 \\ 0 & c_{\theta/2}^2 \end{pmatrix} \tag{54}$$