Gluon Quasi PDF in Large Momentum Effective Theory

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Outline

- Parton Distribution Functions
- > Quasi PDF and LaMET
- Brief Results for quark PDFs
- Gluon quasi PDF: renormalization
- > Summary

Success of the Standard Model(SM)

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	m				ן [fb ^{−1}]	Reference	
nn	$\sigma = 96.07 \pm 0.18 \pm 0.91 \text{ mb (data)}$ COMPETE HPR1R2 (theory)			4	50×10 ⁻⁸	PLB 761 (2016) 158	
PP	$\sigma = 95.35 \pm 0.38 \pm 1.3 \text{ mb} \text{ (data)}$ COMPETE HPR1R2 (theory)	ATLAS Preliminary	¢	0	8×10 ⁻⁸	Nucl. Phys. B, 486-548 (2014)	
w	$\sigma = 190.1 \pm 0.2 \pm 6.4$ nb (data) DYNNLO + CT14NNLO (theory)		. . P	P P	0.081	PLB 759 (2016) 601	
••	$\sigma = 98.71 \pm 0.028 \pm 2.191$ nb (data) DYNNLO + CT14NNLO (theory)	_ Run 1,2 √ <i>s</i> = 7,8,13 TeV	/ Q	P	4.6	EPJC 77 (2017) 367	
-	$\sigma = 34.24 \pm 0.03 \pm 1.00$ lb (data) DYNNLO+CT14 NNLO (theory)		_ ₽		3.2	JHEP 02 (2017) 117	
Z	$\sigma = 29.53 \pm 0.03 \pm 0.77 \text{ nb} (data)$		A	Ê	20.2	JHEP 02 (2017) 117	
	DYNNLO+CT14 NNLO (theory) $\sigma = 818 \pm 8 \pm 35 \text{ pb (data)}$	<u>+</u>	Ŷ		4.6	JHEP 02 (2017) 117	
. .	top++ NNLO+NLL (theory) $\sigma = 242.9 \pm 1.7 \pm 8.6 \text{ pb} \text{ (data)}$, ⁴			3.2	FEB /01 (2010) 130	
ττ	top++ NNLO+NNLL (theory) $\sigma = 182.9 \pm 3.1 \pm 6.4$ pb (data)	4			20.2	EPIC 74: 3109 (2014)	
	$\sigma = 247 \pm 6 \pm 46 \text{ pb (data)}$	Y			4.0	JHEP 04 (2017) 086	1000000
tt chan	$\sigma = 89.6 \pm 1.7 \pm 7.2 - 6.4 \text{ pb} (data)$	× ¹			20.3	EPJC 77 (2017) 531	JCCESS OI
•t=chan	$\sigma = 68 \pm 2 \pm 8 \text{ (heory)}$	5 S		5	4.6	PRD 90, 112006 (2014)	
	$\sigma = 142 \pm 5 \pm 13 \text{ pb (data)}$			6	3.2	PLB 773 (2017) 354	
ww	$\sigma = 68.2 \pm 1.2 \pm 4.6 \text{ pb (data)}$ NNI O (theory)				20.3	PLB 763, 114 (2016)	FVV and
	$\sigma = 51.9 \pm 2 \pm 4.4 \text{ pb (data)}$ NNLO (theory)	۵ ^۲		O	4.6	PRD 87, 112001 (2013) PRL 113, 212001 (2014)	
	$\sigma = 57 + 6 - 5.9 + 4 - 3.3 \text{ pb (data)}$ LHC-HXSWG YR4 (theory)	ò		Ċ.	36.1	ATLAS-CONF-2017-047	CI
н	$\sigma = 27.7 \pm 3 + 2.3 - 1.9 \text{ pb (data)}$ LHC-HXSWG YR4 (theory)	Δ.	_		20.3	EPJC 76, 6 (2016)	tlavor
	$\sigma = 22.1 + 6.7 - 5.3 + 3.3 - 2.7 \text{ pb (data)}$ LHC-HXSWG YR4 (theory)	Þ 📃	Theory		4.5	EPJC 76, 6 (2016)	navoi
	$\sigma = 94 \pm 10 + 28 - 23 \text{ pb (data)}$ NLO+NNLL (theory)				3.2	JHEP 01 (2018) 63	
Wt	$\sigma = 23 \pm 1.3 + 3.4 - 3.7 \text{ pb (data)}$ NLO+NLL (theory)	A L	.HC pp √s = 7 TeV		20.3	JHEP 01, 064 (201	ctors but
	$\sigma = 16.8 \pm 2.9 \pm 3.9 \text{ pb} \text{ (data)}$ NLO+NLL (theory)		Data		2.0	PLB 716, 142-159 (2012)	ciors but
	$\sigma = 51 \pm 0.8 \pm 2.4$ pb (data) MATRIX (NNLO) (theory)	, ¢ 🗖	stat	P P	36.1	ATLAS-CONF-2018-034 PLB 761 (2016) 179	
WZ	$\sigma = 24.3 \pm 0.0 \pm 0.9 \text{ pb} (\text{data})$ MATRIX (NNLO) (theory)	Å	stat ⊕ syst	≜	20.3	PRD 93, 092004 (2016) PLB 761 (2016) 179	((((((((((((((((((((((((((((((((((((
	$\sigma = 19 + 1.4 - 1.5 \pm 1$ pb (data) MATRIX (NNLO) (theory)	QL	.HC pp √s = 8 TeV	P	4.6	PLB 761 (2016) 179	
	$\sigma = 17.3 \pm 0.0 \pm 0.0 \text{ bb} (\text{data})$ Matrix (NNLO) & Sherpa (NLO) (theory) $\sigma = 7.3 \pm 0.4 \pm 0.4 = 0.3 \text{ pb} (\text{data})$, P	Data	_ P	36.1	PRD 97 (2018) 032005	÷
ZZ	$r = 6.7 \pm 0.7 \pm 0.5 \pm 0.4 \text{ pb} (data)$ NNLO (theory)	<u>A</u>	stat	A	20.3	JHEP 01, 099 (2017)	
+	$\sigma = 4.8 \pm 0.8 \pm 1.6 \pm 1.3$ pb (data)	<u> </u>	stat ⊕ syst		4.6	PLB 735 (2014) 311	
L _s -chan	$\sigma = 1.5 \pm 0.72 \pm 0.33 \text{ pb (data)}$	4	.HC pp $\sqrt{s} = 13$ TeV —		20.3	PLB 756, 228-246 (2016)	
tīW	Madgraph5 + aMCNLO (theory) $\sigma = 369 + 86 - 79 \pm 44$ fb (data)		Data		3.2	EPJC // (2017) 40	
	$\sigma = 0.92 \pm 0.29 \pm 0.1 \text{ pb (data)}$	· · · · · · · · · · · · · · · · · · ·	stat		20.3	SHEP 11, 172 (2015)	
tτΖ	Madgraph5 + aMCNLO (theory) $\sigma = 176 + 52 - 48 \pm 24 \text{ fb} \text{ (data)}$	Ψ.	stat ⊕ syst		3.2 20.3	.IHEP 11 172 (2015)	
t7i	HELAC-NLO (theory) $\sigma = 620 \pm 170 \pm 160$ fb (data)	ri .			36.1	PLB 780 (2018) 557	
	NLO+NLL (theory)	••••	الــاسر ۸۸۸ _استنت السنينية		00.1		
1	0^{-4} 10^{-3} 10^{-2} 10^{-1}	1 10^1 10^2 10^3 10^4	10^5 10^6 10^{11}	0510152025	5		
	10 10 10 10	- 10 10 10 10			•		
			σ [DD]	data/theory			

Factorization: Parton Model; PDF

1969年,费曼提出高能质子结构的部分子理论,用 部分子分布函数(或费曼分布)来描述质子物理性质



Factorization theorems:

$$d\sigma \sim \int dx_1 dx_2 * f(x_1) * f(x_2) * C(x_1, x_2, Q)$$

PDF: basic inputs for particle physics at hadron colliders.

Factorization: Parton Model; PDF





0.3

Global Fit of Data



From Jun Gao

PDF From First Principle?

- Fitting Results rely on data
- First-principle calculation can cover regions where experiments cannot constrain so well

• The cost of improving calculations could be much lower than building large experiments.

Gluon PDF

prediction for top pair production



1705.04105v2

大动量分数区域的PDF Uncertainty 对于预言高质量 新物理粒子产生有很大影响

Lattice QCD(K.G.Wilson,1974)

- Numerical simulation in discretized Euclidean space-time
- Finite volume (L should be large)
- Finite lattice spacing (a should be small)



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Tremendous successes in hadron spectroscopy, decay constants, strong coupling, form factors, etc.

PDF (or more general parton physics): Minkowski space, real time infinite momentum frame, on the light-cone

Lattice QCD:

Euclidean space, imaginary time (t=i*tau) Difficulty in time

$$x_E^{\mu} x_E^{\mu} = 0, x_E^{\mu} = (0,0,0,0)$$

Unable to distinguish local operator and light-cone operator

One can form local moments to get rid of the timedependence

- $\langle x^n \rangle = \int f(x) x^n dx$: matrix elements of local operators
- However, one can only calculate lowest few moments in practice.
- Higher moments quickly become noisy.

$$\int_{0}^{1} dx \ x^{n} q(x,\mu) dx = a_{n}(\mu) \propto \left\langle P \left| \overline{\psi}(0) \gamma^{+} \widetilde{i} \widetilde{D}^{+} \cdots \widetilde{i} \widetilde{D}^{+} \psi(0) \right| P \right\rangle$$

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Quasi Parton Distribution Functions and Large Momentum Effective Theory (LaMET)

X. Ji, Phys. Rev. Lett. 110 (2013) 262002 X. Ji, Sci.China Phys.Mech.Astron. 57 (2014) 1407-1412

Much content in the following is taken from Prof. Ji's slides.

Center-of-Mass and Internal Motions : non-relativistic case

• In non-relativistic systems, the COM motion decouples from the internal motion in the sense that the internal dynamics is independent of the COM momentum:

$$H = H_{com} + H_{int}$$



- H_{int} is independent of COM momentum and COM position.
- Wave function of the H-atom is independent of its speed.

Galilei transformation

Center-of-Mass and Internal Motions : relativistic case

• Wave functions in the different frame is related by Lorentz boost:

 $|p\rangle = U(\Lambda(p)) |p=0\rangle$, Λ is related to the boost K_i

• Consider the momentum distribution of the constituent $n(k) = \langle p | a_k^+ a_k | p \rangle$

In relativistic bound state, this becomes a COM momentumdependent quantity,

 $n(k) \rightarrow n(k,p) \text{ or } n_p(k)$

• The internal wave function is frame-dependent (pdependent)!

Computing the momentum dependence

• Studying the momentum dependence of an observable O(p) is in principle possible through commutation relation:

 $[O, K_i] = ...$

However, in relativistic theories, the boost operator K is highly non-trivial, it is interaction-dependent, just like the Hamiltonian.

• Computing the p-dependence of an observable is just as difficult as studying the dynamical evolution.

Asymptotic freedom (AF): large momentun

- QCD is an asymptotic-free theory.
- Once there is a large scale in the problem, such a scale dependence can be studied in pert. theory.
- AF allows to compute the large p-dependence in pert. theory:

$$O(p, a) = C_0(\frac{\mu}{p}, ap)o(\mu) + \frac{c_2}{p^2} + \frac{c_4}{p^4} + \cdots$$

where a is some UV cut-off.

Renormalization group equation

• When power suppressions can be ignored:

$$O(\mathbf{p}, a) \sim C_0(\frac{\mu}{p}, a)o(\mu)$$

p-dependence become RG in pert. theory:

$$\frac{dO(p,a)}{dlnp} \sim \frac{dC_0\left(\frac{\mu}{p},a\right)}{dlnp} O(\mu) \sim \gamma_o(\mu) O(p,a)$$

Anomalous dimension:

$$\gamma_o = \frac{1}{C_0} \frac{dC_0\left(\frac{\mu}{p},a\right)}{dlnp} O$$

Fixed point and parton physics

- The RG equation has a fixed point at $P=\infty$
- This is the infinite momentum limit in which the partons were first introduced by Feynman.

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• Thus the parton physics corresponds to framedependent physical observables at the fixed point of the frame transformations.

This tells us how to calculate the parton physics in QCD !

Quasi PDF



- The distribution at a finite but large P shall be calculable in lattice QCD.
- Since it differs from the standard PDF by simply an infinite P limit, it shall have the same infrared (collinear) physics.
- It shall be related to the standard PDF by a matching factor $Z(\frac{\mu}{p})$ which is perturbatively calculable.

Matching onto Light-cone PDF:

$$\tilde{q}(x,\mu^2,P^z) = \int_{-1}^1 \frac{dy}{|y|} Z\left(\frac{x}{y},\frac{\mu}{P^z}\right) q(y,\mu^2) + \mathcal{O}\left(\Lambda^2/(P^z)^2, M^2/(P^z)^2\right) \ ,$$

- Quasi pdfs: finite but large p^z, from "full theory"
- Light-cone pdfs : $p^z \rightarrow \infty$
- Z: matching coefficient, the difference of the UV physics, can be calculated in perturbation theory.

$$Z(x,\mu/P^z) = \delta(x-1) + \frac{\alpha_s}{2\pi} Z^{(1)}(x,\mu/P^z) + \dots$$

Progress on quasi PDF

- One loop matching for quark (Xiong, Ji, Zhang, Zhao, 2013)
- Renormalization (Ji,Zhang,2014)
- Quasi GPD (Ji ,Schafer, Xiong ,Zhang, 2015)
- Quasi TMD and soft factor subtraction (Ji,Sun,Xiong,Yuan,2015)
- "Lattice cross section" approach (Ma, Qiu, 2014)
- Lattice calculation (Lin, Chen, Cohen, Ji, 2014; Chen, Cohen, Ji, Lin, Zhang, 2016)

- Quasi distribution amplitude of Heavy Quarkonia (Jia, Xiong, 2015)
- Non-dipolar Wilson line (Li,2016)
- diquark spectator model (Gamberg, Kang, Vitev, Xing)
- Matching continuum to lattice (T. Ishikawa, Y.Q. Ma, J.W. Qiu, S.Yoshida, 2016)
- 2017...
- 2018...
 - 2019... Many Progress have been made on quasi PDFs, but I can not discuss all important ones.

Progress on quasi PDF

Lattice Collaboration working on quasi-PDFs:

Lattice Parton Physics Project (LP3) Collaboration

J.W. Chen (National Taiwan U.), T. Ishikawa (T.-D. Lee Institute), L. Jin (U. Connecticut and BNL), R.-Z. Li (Michigan State U.), H.-W. Lin (Michigan State U.), Y.-S. Liu (TDLI), A. Schaefer (U. Regensburg), Y.-B. Yang (Michigan State U.), J.-H. Zhang (U. Regensburg), R. Zhang (Beijing Inst. Theory), and Y. Zhao (MIT), et al.

European Twisted Mass Collaboration (ETMC)

C. Alexandrou (U. Cyprus), M. Constantinou (Temple U.), K.Cichy (Adam Mickiewicz U.), K. Jansen (NIC, DESY), F. Steffens (Bonn U.), et al.

> **DESY, Zeuthen** J. Green, et al.

Brookhaven group

T. Izubuchi, L. Jin, K. Kallidonis, N. Karthik, S. Mukherje, P. Petreczky, C. Schugert, S. Syritsyn.

Progress on quasi PDF

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$$u(x) - d(x) - \bar{u}(-x) + \bar{d}(-x)$$

LP3: 1803.04393

ETMC:1803.02685



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Gluon quasi PDF:

ABSTRACT: We present the most precise value for the Higgs gluon-fusion production mode at the LHC. Our result is based through N³LO in QCD, in an ϵ_{os} CT14 NNLO vhere the **top** g(x.O)/5 finitely heavy, while all other St_{g} larks are mass e all pdf sets finite qua with QCD corrections to the creations of the creations of the creations of the creation of the exactly through \mathbf{NLO} . In additin corrections and at $\frac{\text{CJ15}\left[1\right]^{a}}{\text{three}}$ loop inverse mass of the top-quark : 0.01 0.03 0.1 0.3 effects of threshold, Fesquarina and a solution of the traditional QCD WW, Zhao, 1712.03830 SCET approved by invehicler, www.nzhao, 1808.08247 contributions to all tainty of the cross-section Zhom prossing higher-order Rock estion QCD effects beyond N³L, Ma, and unknown mixed QCD-electrow

giuon-iusion piouucnon mou **Gluon PDF** through $N^{3}LO$ in QCD, in finitely heavy, while all othe with QCD corrections H to the **Higgs Production:** gluon-gluon fusion exactly through NLO. In ad inverse mass of the top-qua Cross sections are calculated by Zürich group at N³LO QCD and NLOEW accuracies [Anastasion:2016ce2] Conduct resumma mĦ=125.09 GeV, √s=13 EET approach, which resur σtal 52 pbof the cross-section fr Total Uncertainty: B98 ff (Gaussian) on d N³LO a **PDF: 1.9%** determine the sensitivity of $\alpha_s: 2.6\%$ (DDT) and and A

quasi PDF for gluon: definition?

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Definition of quasi and light-cone gluon distribution

$$f_{g/H}(x,\mu) = \int \frac{d\xi^{-}}{2\pi x P^{+}} e^{-ix\xi^{-}P^{+}} \langle P|F^{+}_{i}(\xi^{-})W(\xi^{-},0,L_{n^{+}})F^{i+}(0)|P\rangle$$
$$\tilde{f}_{g/H}(x,\mu) = \int \frac{dz}{2\pi x P^{z}} e^{-ixzP^{z}} \langle P|F^{z}_{i}(z)W(z,0,L_{n^{z}})F^{iz}(0)|P\rangle$$

≻ Field Strength Tensor: F

 \succ i sums over transverse directions (i=1,2) or full directions

 \succ W(z1,z2, C) is a Wilson line along contour C.

Renormalization of gluon PDF: Linear Divergences



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- Light-cone: n²=0, no linear power divergence;
- Quasi: n²=-1, the integral contributes a linear power divergence! $\frac{dk_0 d^2 k_T * k^4}{k_0 k_0} \sim k$

Lattice Regularization?

Renormalization of gluon PDF: 30 Auxiliary Field Gervais and Neveu, 1980

Wilson line $W(z_1, z_2; C) = \langle \mathcal{Z}(\lambda_1) \overline{\mathcal{Z}}(\lambda_2) \rangle_z$

Gauge invariant non-local operators pairs of gauge invariant composite local operators

$$F^{a}_{\mu\nu}(z_{1})W_{ab}(z_{1}, z_{2}; C)F^{b}_{\rho\sigma}(z_{2}) = \langle (F^{a}_{\mu\nu}(z_{1})\mathcal{Z}_{a}(\lambda_{1})) | \overline{(\mathcal{Z}_{b}(\lambda_{2})}F^{b}_{\rho\sigma}(z_{2}) \rangle \\ = \Omega^{(1)}_{\mu\nu}(z_{1})\overline{\Omega^{(1)}_{\rho\sigma}}(z_{2})$$

$$\Omega_{\mu\nu}^{(1)}(z_1) = F_{\mu\nu}^a(z_1)\mathcal{Z}_a(\lambda_1))$$

Renormalization of gluon PDF: One Loop diagrams



$$\begin{split} I_{1} &= \frac{\alpha_{s}C_{A}}{\pi} \Big\{ \frac{1}{4-d} (A_{a}^{\nu}n^{\mu} - A_{a}^{\mu}n^{\nu})n \cdot \partial \mathcal{Z}_{a}/n^{2} \left(\frac{\pi\mu}{3-d} (n^{\mu}A_{a}^{\nu} - n^{\nu}A_{a}^{\mu})\mathcal{Z}_{a} + reg. \Big\}, \\ I_{2} &= \frac{\alpha_{s}C_{A}}{\pi} \Big\{ \frac{1}{4-d} \Big[\frac{1}{4}F_{a}^{\mu\nu}\mathcal{Z}_{a} + \frac{1}{2} \big(F_{a}^{\mu\rho}n_{\nu}n_{\rho} - F_{a}^{\nu\rho}n_{\mu}n_{\rho}\big)/n^{2} + \frac{1}{2} (A_{a}^{\mu}n^{\nu} - A_{a}^{\nu}n^{\mu})n \cdot \partial \mathcal{Z}_{a}/n^{2} \Big] \\ &+ \frac{\pi\mu}{3-d} (n^{\mu}A_{a}^{\nu} - n^{\nu}A_{a}^{\mu})\mathcal{Z}_{a} + reg. \Big\}, \end{split}$$

No power divergence!

Renormalization of gluon quasi-PDF

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Three operators with the same quantum number

 $\Omega_{\mu\nu}^{(1)} = F_{\mu\nu}^{a} \mathcal{Z}_{a},$ $\Omega_{\mu\nu}^{(2)} = \Omega_{\mu\alpha}^{(1)} \frac{\dot{x}_{\alpha} \dot{x}_{\nu}}{\dot{x}^{2}} - \Omega_{\nu\alpha}^{(1)} \frac{\dot{x}_{\alpha} \dot{x}_{\mu}}{\dot{x}^{2}},$ $\Omega_{\mu\nu}^{(3)} = |\dot{x}|^{-2} (\dot{x}_{\mu} A_{\nu}^{a} - \dot{x}_{\nu} A_{\mu}^{a}) (D\mathcal{Z})_{a},$



Renormalization of gluon quasi-PDF

Different components are renormalized differently!

$$\begin{pmatrix} \Omega_{1,R}^{z\mu} \\ \Omega_{3,R}^{z\mu} \end{pmatrix} = \begin{pmatrix} Z_{22} & Z_{13} \\ 0 & Z_{33} \end{pmatrix} \begin{pmatrix} \Omega_{1}^{z\mu} \\ \Omega_{3}^{z\mu} \end{pmatrix};$$

 $\Omega_{1,R}^{ti} = Z_{11}\Omega_1^{ti}$

Renormalization of gluon PDF: Multiplicatively Renormalizable Operators 34

$$O^{(1)}(z_1, z_2) \equiv F^{ti}(z_1)L(z_1, z_2)F_i^{\ t}(z_2),$$

$$O^{(2)}(z_1, z_2) \equiv F^{zi}(z_1)L(z_1, z_2)F_i^{\ z}(z_2),$$

$$O^{(3)}(z_1, z_2) \equiv F^{ti}(z_1)L(z_1, z_2)F_i^{\ z}(z_2),$$

$$O^{(4)}(z_1, z_2) \equiv F^{z\mu}(z_1)L(z_1, z_2)F_\mu^{\ z}(z_2),$$

Different components are renormalized differently!

First Lattice Simulation



Fan, Yang, Anthony, Lin, Liu, 1808.02077

$$\begin{split} \tilde{H}_0(z,P_z) &= \langle P|\mathcal{O}_0(z)|P\rangle,\\ \mathcal{O}_0 &\equiv \frac{P_0\left(\mathcal{O}(F^t_\mu,F^{\mu t};z) - \frac{1}{4}g^{tt}\mathcal{O}(F^\mu_\nu,F^\nu_\mu;z)\right)}{\frac{3}{4}P_0^2 + \frac{1}{4}P_z^2} \end{split}$$

Summary

LaMET: Parton physics demands new ideas to solve nonperturbative QCD.

Gluon Quasi PDF: Renormalizability; RI/MOM subtraction(O₃); Factorization; One-loop matching; polarized PDF; Mixing on the lattice; BRST/ghost on lattice (p^2/epsilon);

In 5~10 years, expect:

- ✓ Lattice calculation of quark PDFs: 10%
- ✓ Better constraints x~1
- ✓ Distributions: gluon, TMD, GPD

Thank you very much!