Gluon Quasi PDF
in
Large Momentum Effective Theory

Wei Wang
Shanghai Jiao Tong University

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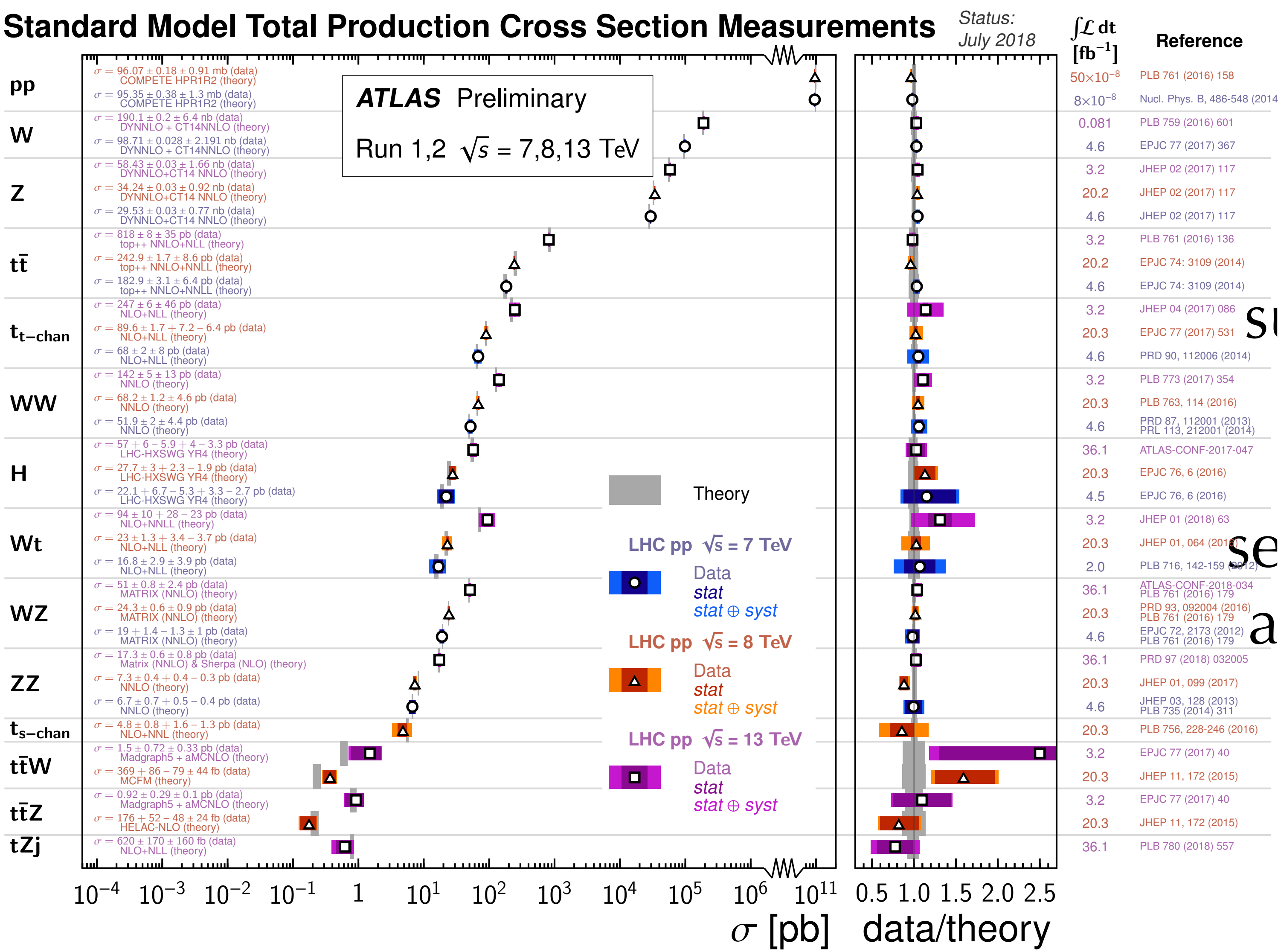
Outline

- **Parton Distribution Functions**
- **Quasi PDF and LaMET**
- **Brief Results for quark PDFs**
- **Gluon quasi PDF: renormalization**
- **Summary**

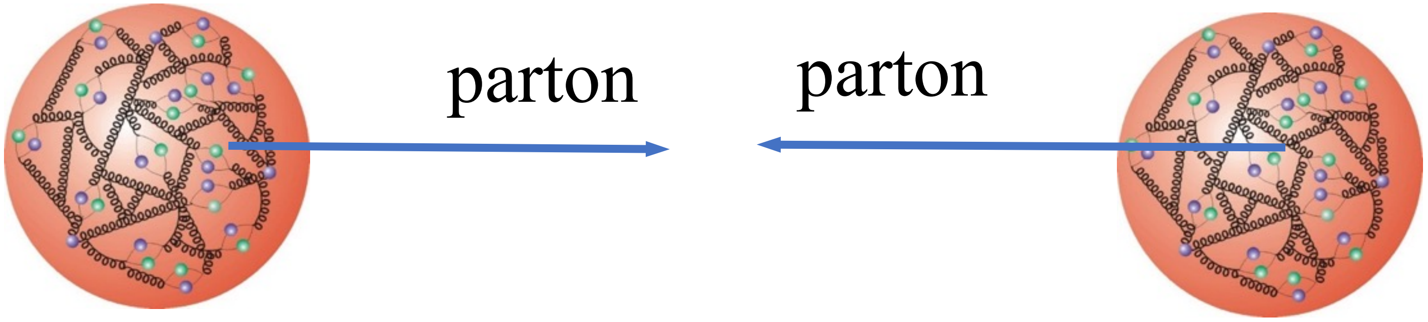
Success of the Standard Model(SM)



success of EW and flavor sectors but also QCD

Factorization: Parton Model; PDF

1969年，费曼提出高能质子结构的部分子理论，用部分子分布函数（或费曼分布）来描述质子物理性质



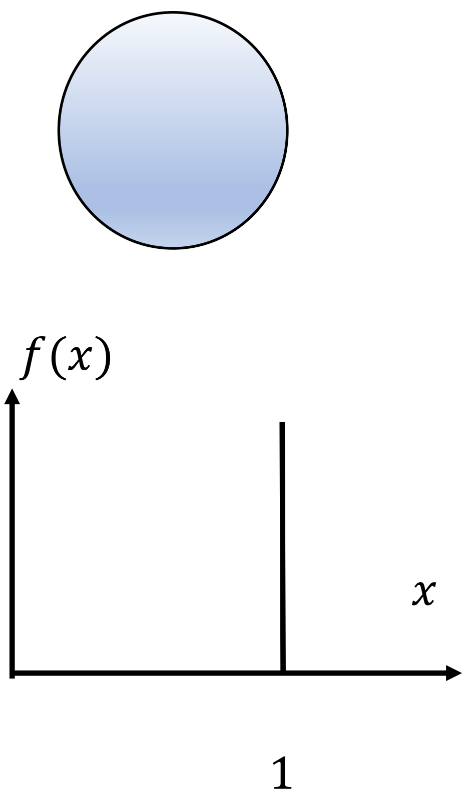
Factorization theorems:

$$d\sigma \sim \int dx_1 dx_2 * f(x_1) * f(x_2) * C(x_1, x_2, Q)$$

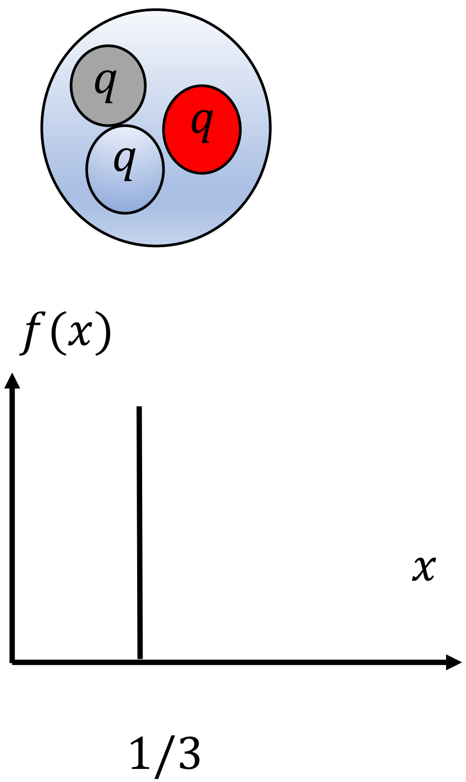
PDF: basic inputs for particle physics at hadron colliders.

Factorization: Parton Model; PDF

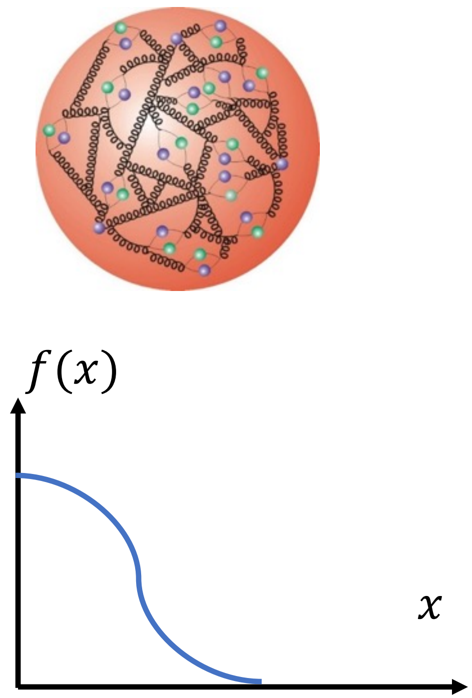
Elastic



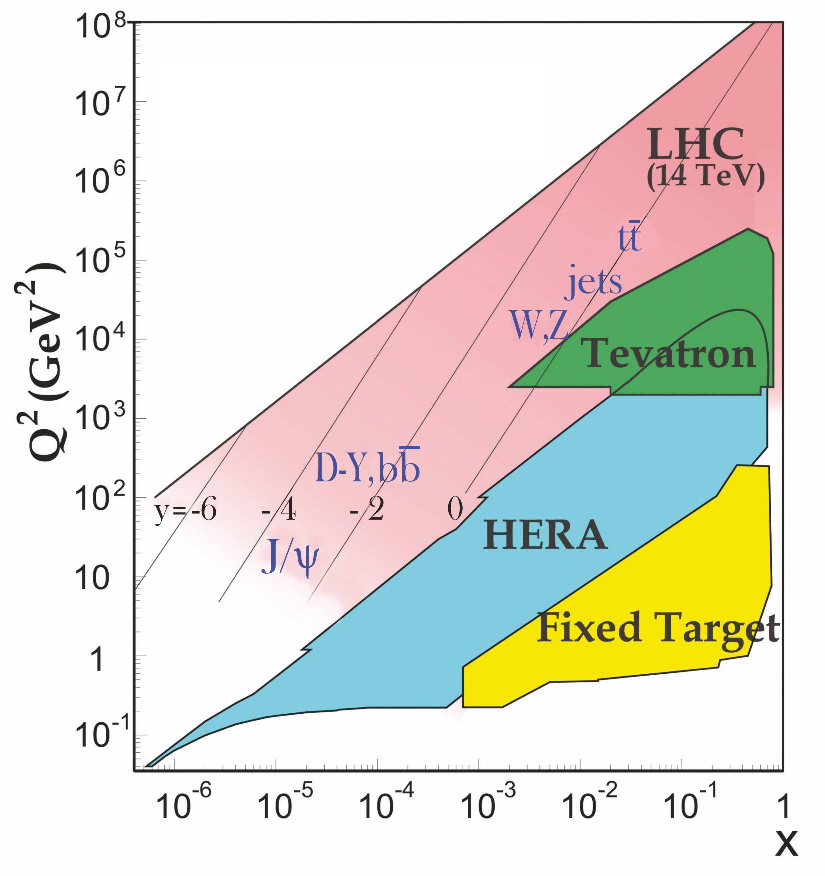
3 free quarks

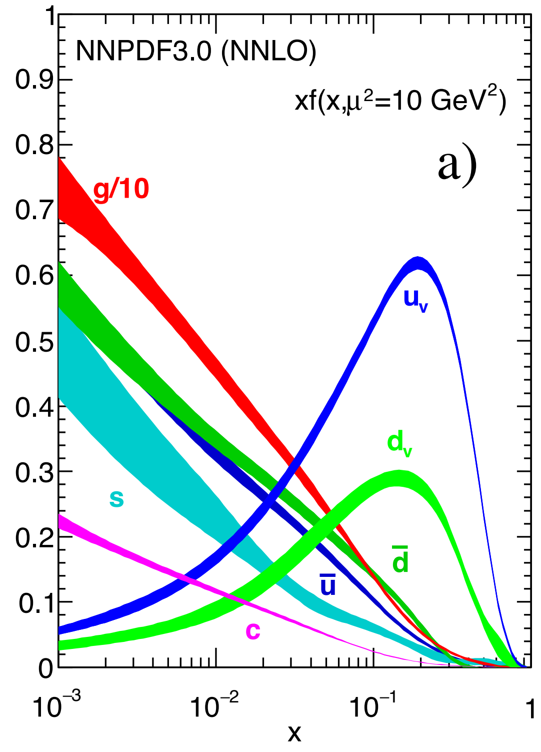


Quarks and Gluons
Many body problem

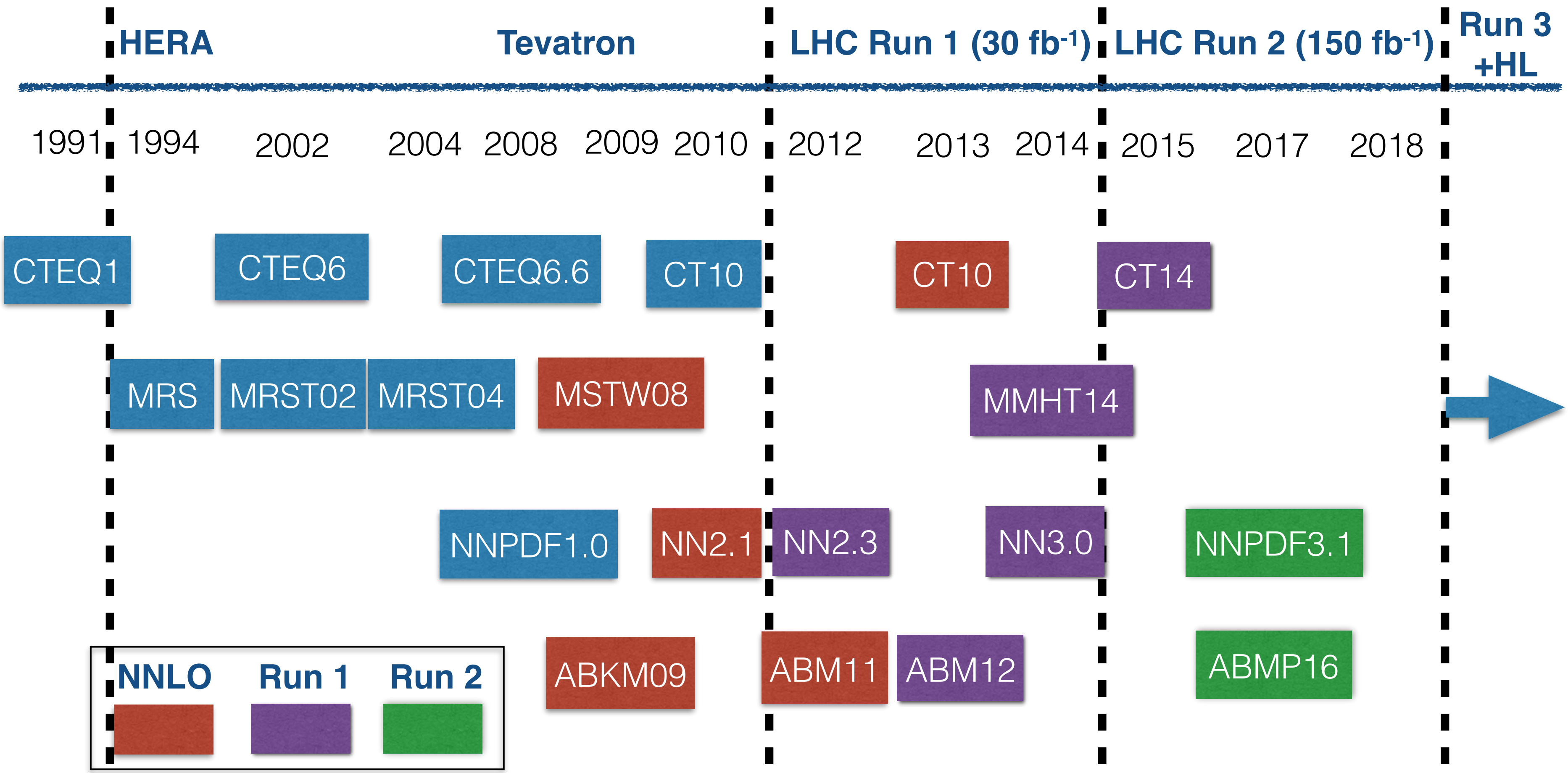


Global Fit of Data





Global Fit of Data



From Jun Gao

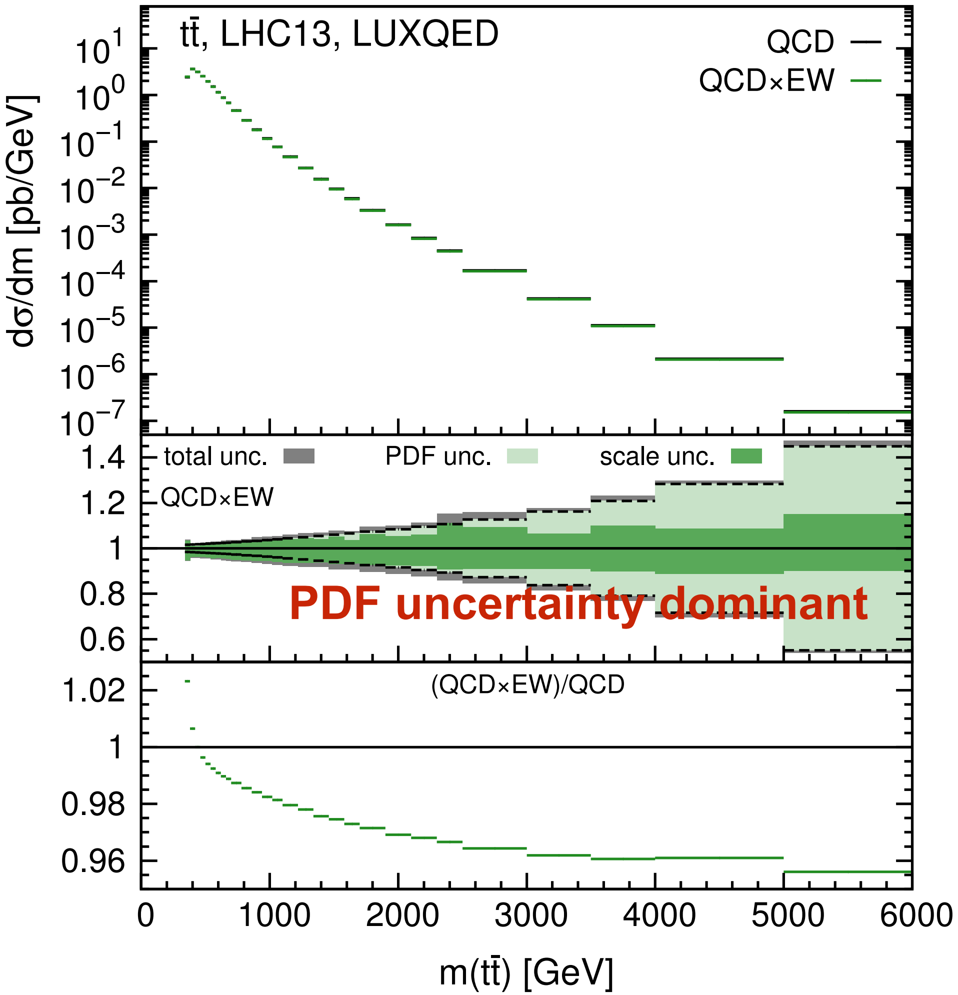
PDF From First Principle?

- **Fitting Results rely on data**

- **First-principle calculation can cover regions where experiments cannot constrain so well**

- **The cost of improving calculations could be much lower than building large experiments.**

Gluon PDF

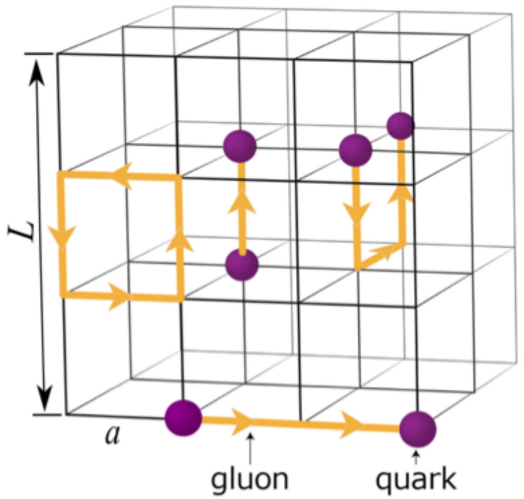


1705.04105v2

大动量分数区域的PDF Uncertainty 对于预言高质量新物理粒子产生有很大影响

Lattice QCD(K.G.Wilson,1974)

- Numerical simulation in discretized Euclidean space-time
- Finite volume (L should be large)
- Finite lattice spacing (a should be small)



Tremendous successes in hadron spectroscopy, decay constants, strong coupling, form factors, etc.

Lattice QCD: PDF?

PDF (or more general parton physics):
Minkowski space, real time
infinite momentum frame, on the light-cone

Lattice QCD:

Euclidean space, imaginary time (t=i*tau)
Difficulty in time

$$x_E^\mu \, x_E^\mu = 0, \; x_E^\mu = (0,0,0,0)$$

Unable to distinguish local operator and light-cone operator

Lattice QCD: PDF?

One can form local moments to get rid of the time-dependence

- $\langle x^n \rangle = \int f(x) x^n dx$: matrix elements of local operators
- However, one can only calculate lowest few moments in practice.
- Higher moments quickly become noisy.

$$\int_0^1 dx\ x^n q(x,\mu) dx = a_n(\mu) \propto \left\langle P \middle| \bar{\psi}(0) \gamma^+ \overbrace{i\overleftrightarrow{D}^+ \cdots i\overleftrightarrow{D}^+}^{n} \psi(0) \middle| P \right\rangle$$

Quasi Parton Distribution Functions
and
Large Momentum Effective Theory
(LaMET)

X. Ji, Phys. Rev. Lett. 110 (2013) 262002
X. Ji, Sci.China Phys.Mech.Astron. 57 (2014) 1407-1412

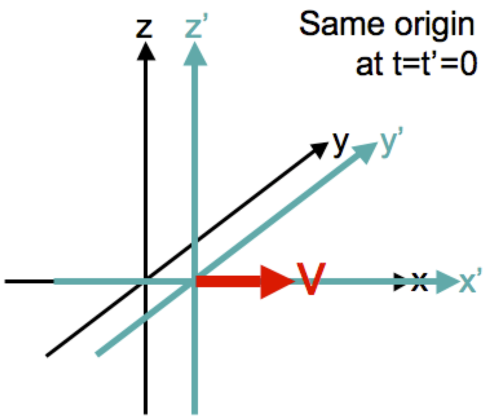
Much content in the following is taken from Prof. Ji's slides.

Center-of-Mass and Internal Motions : non-relativistic case

- In non-relativistic systems, the COM motion decouples from the internal motion in the sense that the internal dynamics is independent of the COM momentum:

$$H = H_{com} + H_{int}$$

- H_{int} is independent of COM momentum and COM position.
- Wave function of the H-atom is independent of its speed.



Galilei transformation

Center-of-Mass and Internal Motions : relativistic case

- Wave functions in the different frame is related by Lorentz boost:

$|p\rangle = U(\Lambda(p))\ |p{=}0\rangle$, Λ is related to the boost K_i

- Consider the momentum distribution of the constituent

$$n(k) = \langle p|a_k^+ a_k|p\rangle$$

In relativistic bound state, this becomes a COM momentum-dependent quantity,

$$n(k) \rightarrow n(k,p) \text{ or } n_p(k)$$

- The internal wave function is frame-dependent (p-dependent)!

Computing the momentum dependence

- Studying the momentum dependence of an observable O(p) is in principle possible through commutation relation:

$$[O, K_i] = \ldots$$

However, in relativistic theories, the boost operator K is highly non-trivial, it is interaction-dependent, just like the Hamiltonian.

- Computing the p-dependence of an observable is just as difficult as studying the dynamical evolution.

Asymptotic freedom (AF): large momentum

- QCD is an asymptotic-free theory.
- Once there is a large scale in the problem, such a scale dependence can be studied in pert. theory.
- AF allows to compute the large p-dependence in pert. theory:

$$O(\mathrm{p},a) = \mathrm{C}_0\left(\frac{\mu}{p}, a\mathrm{p}\right) o(\mu) + \frac{c_2}{p^2} + \frac{c_4}{p^4} + \cdots$$

where a is some UV cut-off.

Renormalization group equation

- When power suppressions can be ignored:

$$O(\mathrm{p}, a) \sim \mathrm{C}_0(\frac{\mu}{P}, a)o(\mu)$$

p-dependence become RG in pert. theory:

$$\frac{dO(p,a)}{dlnp} \sim \frac{dC_0\left(\frac{\mu}{p},a\right)}{dlnp} O(\mu) \sim \gamma_o(\mu)\ O(p,a)$$

Anomalous dimension:

$$\gamma_o = \frac{1}{C_0}\frac{dC_0\left(\frac{\mu}{p},a\right)}{dlnp} O$$

Fixed point and parton physics

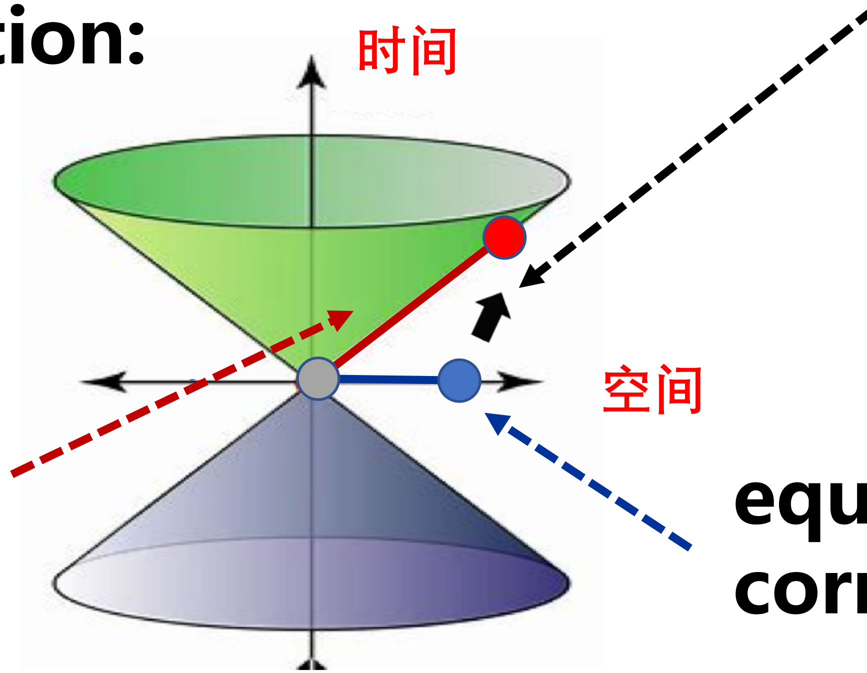
- The RG equation has a fixed point at $P=\infty$
- This is the infinite momentum limit in which the partons were first introduced by Feynman.
- Thus the parton physics corresponds to frame-dependent physical observables at the fixed point of the frame transformations.

This tells us how to calculate the parton physics in QCD！

Quasi PDF

$$\tilde{q}(x, \mu^2, P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P|\overline{\psi}(z)\gamma^z \times \exp\left(-ig\int_0^z dz' A^z(z')\right)\psi(0)|P\rangle$$

Frame transformation:



equal time correlation

PDF: light-cone correlation

Finite but large P

- The distribution at a finite but large P shall be calculable in lattice QCD.

- Since it differs from the standard PDF by simply an infinite P limit, it shall have the same infrared (collinear) physics.

- It shall be related to the standard PDF by a matching factor $Z(\frac{\mu}{P})$ which is perturbatively calculable.

Quasi PDF

Matching onto Light-cone PDF:

$$\tilde{q}(x,\mu^2,P^z) = \int_{-1}^{1} \frac{dy}{|y|} Z\left(\frac{x}{y},\frac{\mu}{P^z}\right) q(y,\mu^2) + \mathcal{O}\left(\Lambda^2/(P^z)^2, M^2/(P^z)^2\right) \ ,$$

- Quasi pdfs: finite but large p^z, from “full theory”
- Light-cone pdfs : $p^z \to \infty$
- Z: matching coefficient, the difference of the UV physics, can be calculated in perturbation theory.

$$Z(x,\mu/P^z) = \delta(x-1) + \frac{\alpha_s}{2\pi} Z^{(1)}(x,\mu/P^z) + \ldots$$

Progress on quasi PDF

- One loop matching for quark (Xiong, Ji, Zhang, Zhao,2013)
- Renormalization (Ji,Zhang,2014)
- Quasi GPD (Ji ,Schafer, Xiong ,Zhang, 2015)
- Quasi TMD and soft factor subtraction (Ji,Sun,Xiong,Yuan,2015)
- "Lattice cross section" approach (Ma, Qiu, 2014)
- Lattice calculation (Lin, Chen, Cohen,Ji, 2014; Chen, Cohen, Ji, Lin , Zhang ,2016)
- Quasi distribution amplitude of Heavy Quarkonia (Jia, Xiong,2015)
- Non-dipolar Wilson line (Li,2016)
- diquark spectator model (Gamberg, Kang, Vitev, Xing)
- Matching continuum to lattice (T. Ishikawa, Y.Q. Ma, J.W. Qiu, S.Yoshida, 2016)
- 2017…
- 2018…
- 2019…

Many Progress have been made on quasi PDFs, but I can not discuss all important ones.

Progress on quasi PDF

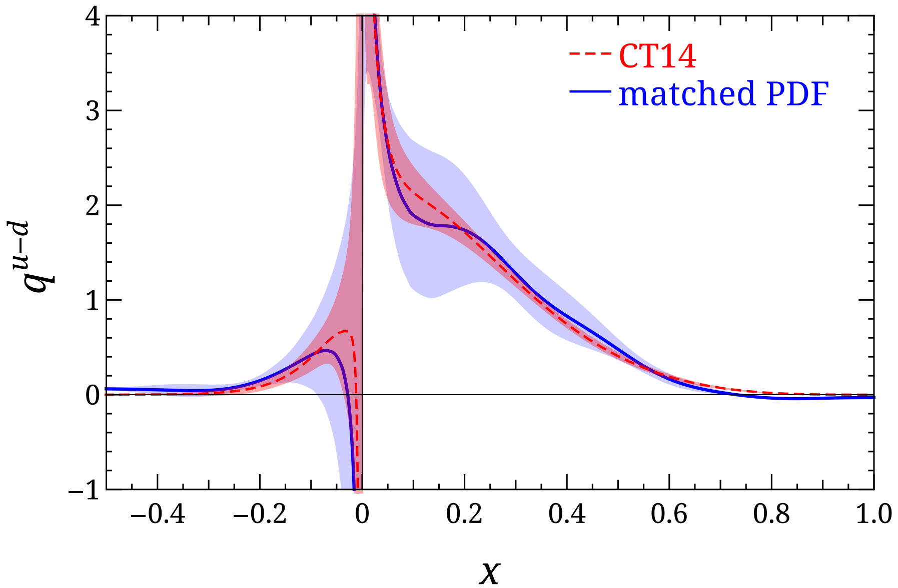
Lattice Collaboration working on quasi-PDFs:

- **Lattice Parton Physics Project (LP3) Collaboration**
  J.W. Chen (National Taiwan U.), T. Ishikawa (T.-D. Lee Institute), L. Jin (U. Connecticut and BNL), R.-Z. Li (Michigan State U.), H.-W. Lin (Michigan State U.), Y.-S. Liu (TDLI), A. Schaefer (U. Regensburg), Y.-B. Yang (Michigan State U.), J.-H. Zhang (U. Regensburg), R. Zhang (Beijing Inst. Theory), and Y. Zhao (MIT), et al.
- **European Twisted Mass Collaboration (ETMC)**
  C. Alexandrou (U. Cyprus) , M. Constantinou (Temple U.), K.Cichy (Adam Mickiewicz U.), K. Jansen (NIC, DESY), F. Steffens (Bonn U.), et al.
- **DESY, Zeuthen** J. Green, et al.
- **Brookhaven group**
  T. Izubuchi, L. Jin, K. Kallidonis, N. Karthik, S. Mukherje, P. Petreczky, C. Schugert, S. Syritsyn.

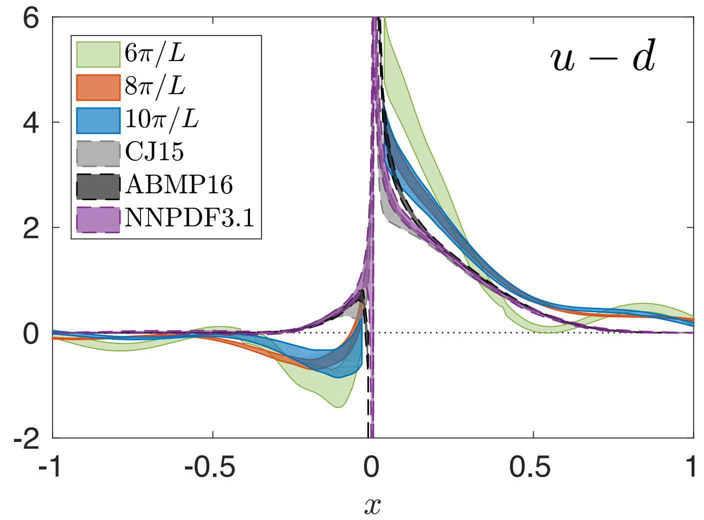
Progress on quasi PDF

$$u(x) - d(x) - \bar{u}(-x) + \bar{d}(-x)$$

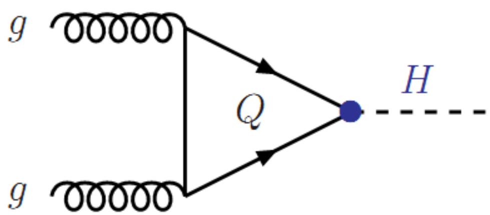
LP3: 1803.04393

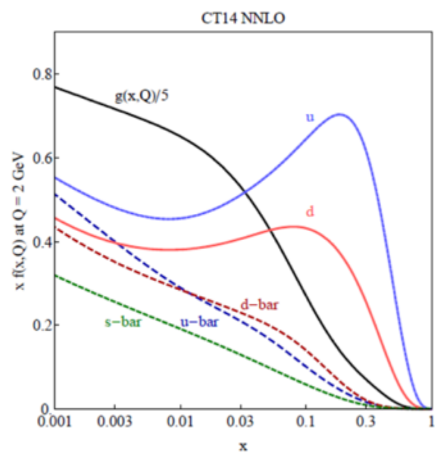


ETMC:1803.02685



Gluon quasi PDF:
Definition and Renormalization





WW,Zhao,Zhu,1708.02458

WW, Zhao, 1712.03830

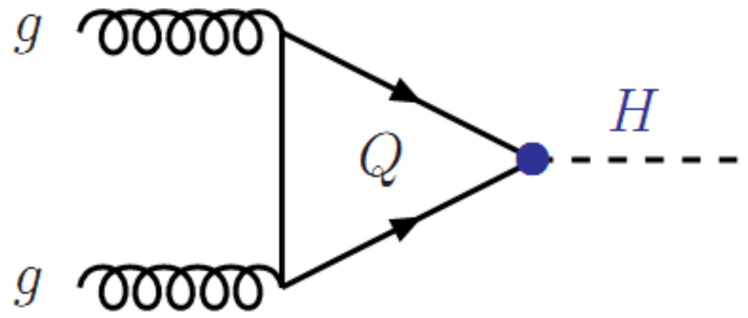
Zhang, Ji, Schafer, WW, Zhao,1808.10824

WW, Zhang, Zhao, Zhu, in preparation

See also Li, Ma, Qiu, 1809.01836

Gluon PDF

Higgs Production:
gluon-gluon fusion



Cross sections are calculated by Zürich group at N^3LO QCD and NLO EW accuracies [Anastasiou:2016cez]

mH=125.09 GeV, $\sqrt{s}$=13 TeV

σ=48.52pb

Total Uncertainty: 3.9% (Gaussian)
PDF: 1.9%
α_s: 2.6%

quasi PDF for gluon: definition?

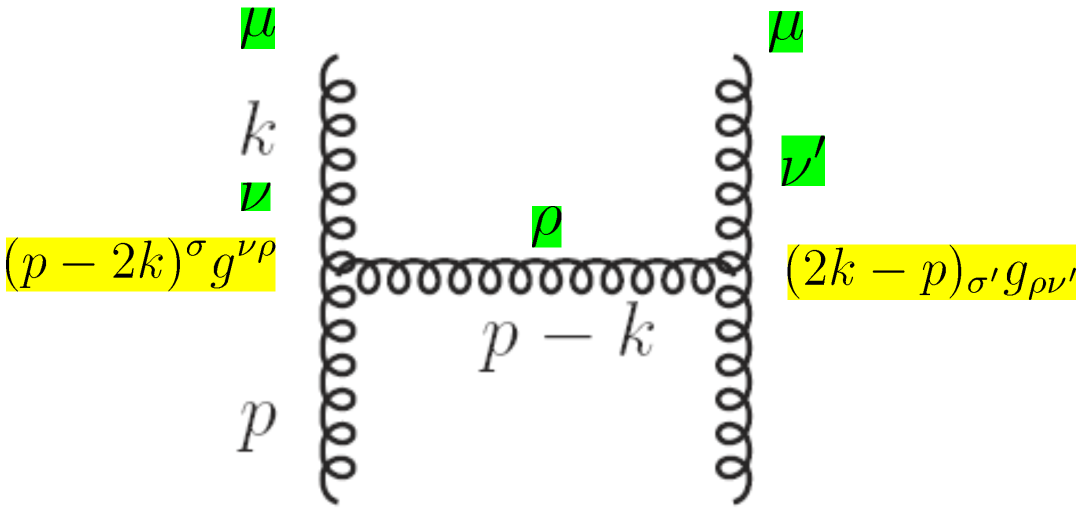
Definition of quasi and light-cone gluon distribution

$$f_{g/H}(x,\mu) = \int \frac{d\xi^-}{2\pi x P^+} e^{-ix\xi^- P^+} \langle P|F^+{}_i(\xi^-)W(\xi^-,0,L_{n^+})F^{i+}(0)|P\rangle$$

$$\tilde{f}_{g/H}(x,\mu) = \int \frac{dz}{2\pi x P^z} e^{-ixzP^z} \langle P|F^z{}_i(z)W(z,0,L_{n^z})F^{iz}(0)|P\rangle$$

- Field Strength Tensor: F
- i sums over transverse directions (i=1,2) or full directions
- W(z1,z2, C) is a Wilson line along contour C.

Renormalization of gluon PDF: Linear Divergences



$\propto n^2$

- Light-cone: $n^2=0$, no linear power divergence;
- Quasi: $n^2=-1$, the integral contributes a linear power divergence!

$$\frac{dk_0 d^2k_T * k^4}{k^6} \sim k$$

Lattice Regularization?

Renormalization of gluon PDF: Auxiliary Field

Gervais and Neveu, 1980

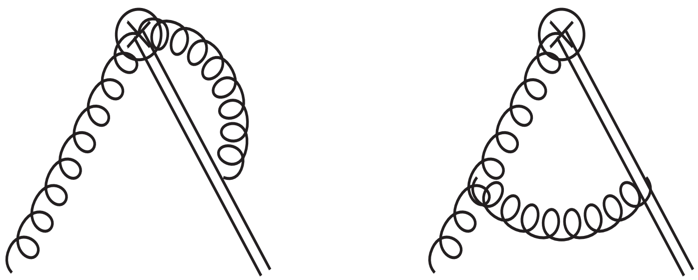
Wilson line $W(z_1, z_2; C) = \langle \mathcal{Z}(\lambda_1)\bar{\mathcal{Z}}(\lambda_2)\rangle_z$

Gauge invariant non-local operators
pairs of gauge invariant composite local operators

$$F^a_{\mu\nu}(z_1)W_{ab}(z_1, z_2; C)F^b_{\rho\sigma}(z_2) = \left\langle (F^a_{\mu\nu}(z_1)\mathcal{Z}_a(\lambda_1)) \middle| \overline{(\mathcal{Z}_b}(\lambda_2)F^b_{\rho\sigma}(z_2)) \right\rangle$$
$$= \Omega^{(1)}_{\mu\nu}(z_1)\overline{\Omega^{(1)}_{\rho\sigma}}(z_2)$$

$$\Omega^{(1)}_{\mu\nu}(z_1) = F^a_{\mu\nu}(z_1)\mathcal{Z}_a(\lambda_1))$$

Renormalization of gluon PDF: One Loop diagrams



$$I_1 = \frac{\alpha_s C_A}{\pi}\Big\{\frac{1}{4-d}(A_a^\nu n^\mu - A_a^\mu n^\nu) n\cdot\partial \mathcal{Z}_a/n^2 - \frac{\pi\mu}{3-d}(n^\mu A_a^\nu - n^\nu A_a^\mu)\mathcal{Z}_a + reg.\Big\},$$

$$I_2 = \frac{\alpha_s C_A}{\pi}\Big\{\frac{1}{4-d}\Big[\frac{1}{4}F_a^{\mu\nu}\mathcal{Z}_a + \frac{1}{2}\big(F_a^{\mu\rho}n_\nu n_\rho - F_a^{\nu\rho}n_\mu n_\rho\big)/n^2 + \frac{1}{2}(A_a^\mu n^\nu - A_a^\nu n^\mu) n\cdot\partial\mathcal{Z}_a/n^2\Big] + \frac{\pi\mu}{3-d}(n^\mu A_a^\nu - n^\nu A_a^\mu)\mathcal{Z}_a + reg.\Big\},$$

No power divergence!

Renormalization of gluon quasi-PDF

Three operators with the same quantum number

$$\Omega^{(1)}_{\mu\nu} = F^a_{\mu\nu}\mathcal{Z}_a,$$
$$\Omega^{(2)}_{\mu\nu} = \Omega^{(1)}_{\mu\alpha}\frac{\dot{x}_\alpha\dot{x}_\nu}{\dot{x}^2} - \Omega^{(1)}_{\nu\alpha}\frac{\dot{x}_\alpha\dot{x}_\mu}{\dot{x}^2},$$
$$\Omega^{(3)}_{\mu\nu} = |\dot{x}|^{-2}(\dot{x}_\mu A^a_\nu - \dot{x}_\nu A^a_\mu)(D\mathcal{Z})_a,$$

$$\begin{pmatrix}\Omega^{\mu\nu}_{1,R}\\ \Omega^{\mu\nu}_{2,R}\\ \Omega^{\mu\nu}_{3,R}\end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{22}-Z_{11} & Z_{13}\\ 0 & Z_{22} & Z_{13}\\ 0 & 0 & Z_{33}\end{pmatrix}\begin{pmatrix}\Omega^{\mu\nu}_{1}\\ \Omega^{\mu\nu}_{2}\\ \Omega^{\mu\nu}_{3}\end{pmatrix}.$$

Renormalization of gluon quasi-PDF

Different components are renormalized differently!

$$\begin{pmatrix} \Omega^{z\mu}_{1,R} \\ \Omega^{z\mu}_{3,R} \end{pmatrix} = \begin{pmatrix} Z_{22} & Z_{13} \\ 0 & Z_{33} \end{pmatrix} \begin{pmatrix} \Omega^{z\mu}_{1} \\ \Omega^{z\mu}_{3} \end{pmatrix};$$

$$\Omega^{ti}_{1,R} = Z_{11}\Omega^{ti}_{1}$$

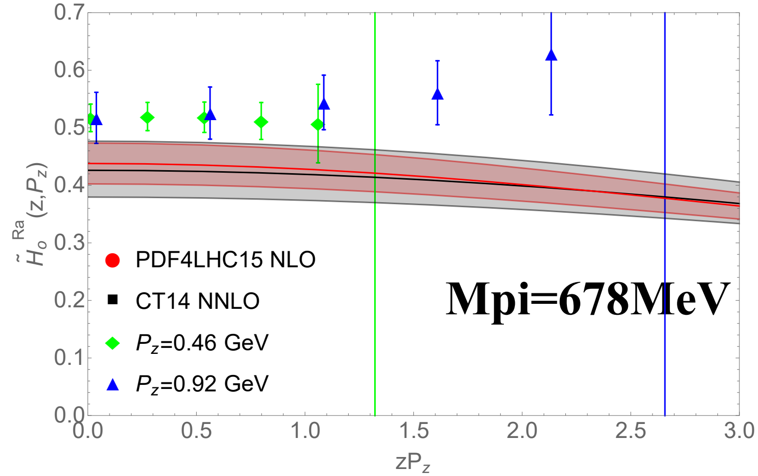
Renormalization of gluon PDF:

Multiplicatively Renormalizable Operators

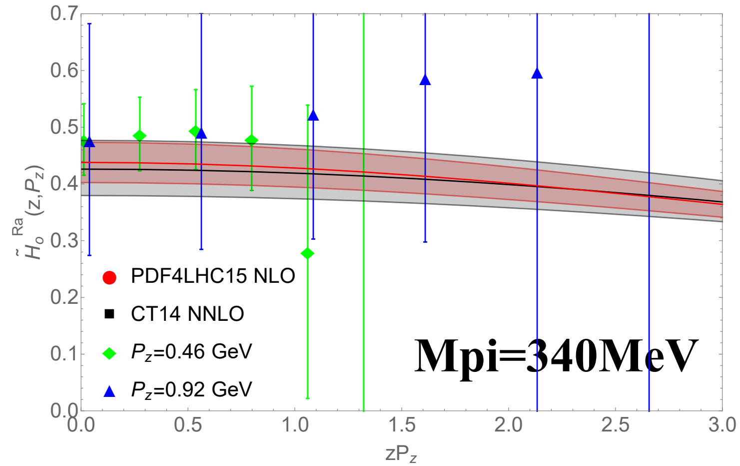
$$O^{(1)}(z_1, z_2) \equiv F^{ti}(z_1)L(z_1, z_2)F_i{}^t(z_2),$$
$$O^{(2)}(z_1, z_2) \equiv F^{zi}(z_1)L(z_1, z_2)F_i{}^z(z_2),$$
$$O^{(3)}(z_1, z_2) \equiv F^{ti}(z_1)L(z_1, z_2)F_i{}^z(z_2),$$
$$O^{(4)}(z_1, z_2) \equiv F^{z\mu}(z_1)L(z_1, z_2)F_\mu{}^z(z_2),$$

Different components are renormalized differently!

First Lattice Simulation



Fan, Yang, Anthony, Lin, Liu,1808.02077



$$\tilde{H}_0(z, P_z) = \langle P|\mathcal{O}_0(z)|P\rangle,$$

$$\mathcal{O}_0 \equiv \frac{P_0\left(\mathcal{O}(F^t_\mu, F^{\mu t}; z) - \frac{1}{4}g^{tt}\mathcal{O}(F^\mu_\nu, F^\nu_\mu; z)\right)}{\frac{3}{4}P_0^2 + \frac{1}{4}P_z^2}.$$

Summary

LaMET: Parton physics demands new ideas to solve non-perturbative QCD.

Gluon Quasi PDF:

Renormalizability; RI/MOM subtraction(O_3);

Factorization; One-loop matching; polarized PDF;

Mixing on the lattice; BRST/ghost on lattice (p^2/epsilon);

In 5~10 years, expect:

- ✓ Lattice calculation of quark PDFs: 10%
- ✓ Better constraints x~1
- ✓ Distributions: gluon, TMD, GPD

Thank you very much!