用格点QCD计算稀有衰变

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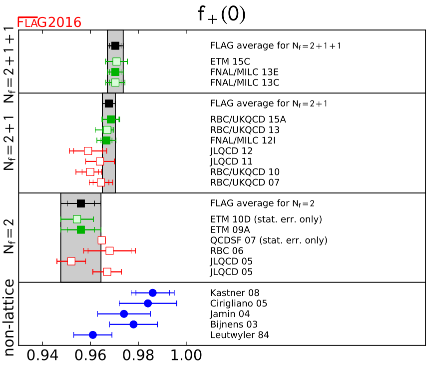
01/25/2019，粒子物理前沿问题研讨会 @ 中山大学

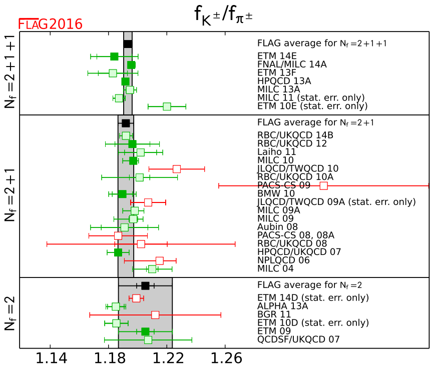
N. Christ, XF, G. Martinelli, C. Sachrajda, PRD91 (2015) 114510
N. Christ, XF, A. Portelli, C. Sachrajda, PRD93 (2016) 114517
Z. Bai, N. Christ, XF, A. Lawson, A. Portelli, C. Sachrajda, PRL118 (2017) 252001
Z. Bai, N. Christ, XF, A. Lawson, A. Portelli, C. Sachrajda, PRD98 (2018) 074509
XF, L. Jin, X. Tuo, S. Xia, PRL122 (2019) 022001

Flavor Lattice Averaging Group (FLAG) average, updated in Nov. 2016

$$f_+^{K\pi}(0) = 0.9706(27) \quad \Rightarrow \quad 0.28\% \text{ error}$$

$$f_{K^\pm}/f_{\pi^\pm} = 1.1933(29) \quad \Rightarrow \quad 0.25\% \text{ error}$$





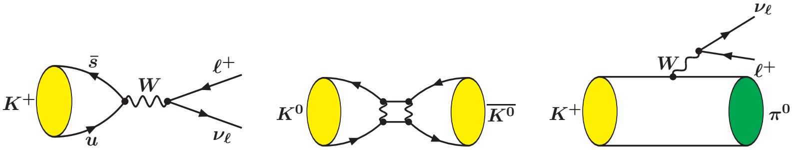
格点输入+实验测量 [arXiv:1411.5252, 1509.02220]

$$K_{\ell 3} \quad \Rightarrow \quad |V_{us}| f_+(0) = 0.2165(4) \quad \Rightarrow \quad |V_{us}| = 0.2231(7)$$

$$K_{\mu 2}/\pi_{\mu 2} \quad \Rightarrow \quad \left|\frac{V_{us}}{V_{ud}}\right| \frac{f_{K^\pm}}{f_{\pi^\pm}} = 0.2760(4) \quad \Rightarrow \quad \left|\frac{V_{us}}{V_{ud}}\right| = 0.2313(7)$$

“常规”物理量和“非常规”物理量

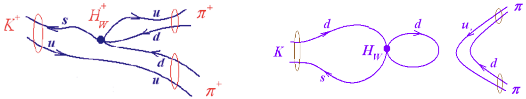
格点 QCD 研究的“常规”物理量:



- 初态、末态仅包含单个强子，或者直接由真空态给出
- 单个强子不能是强衰变共振态粒子(本质还是多强子系统)
- 初末态粒子所携带的动量远小于 $1/a$
- 局域强子矩阵元

突破“常规”，比如

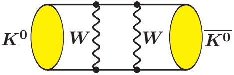
- $K \to \pi\pi$ decay: $\langle \pi\pi | H_W | K \rangle$

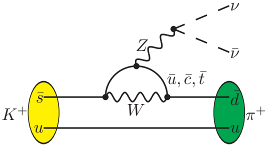


- 长程物理过程中的强相互作用贡献

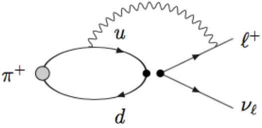
长程过程与非局域强子矩阵元$\langle f|O_1\, O_2|i\rangle$

高阶电弱相互作用

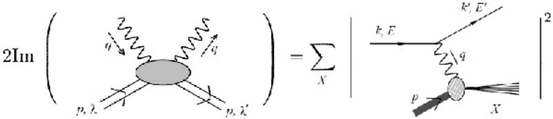




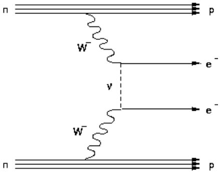
电磁修正效应



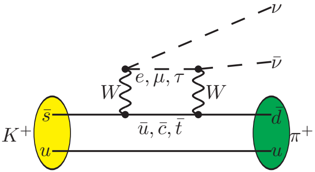
Inclusive decay 和深度非弹散射

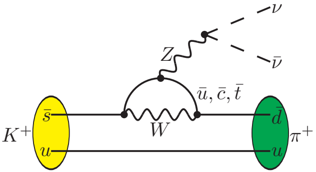


Neutrinoless double beta decay

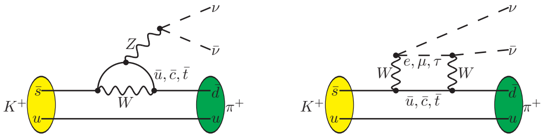


以 $K^+ \to \pi^+ \nu\bar{\nu}$ 衰变为例





$K^+ \to \pi^+\nu\bar{\nu}$: 实验 vs 标准模型



Z-exchange　　　　W-W

$K^+ \to \pi^+\nu\bar{\nu}$: 最大的贡献来自 top quark，所以理论上很干净

$$\mathcal{H}_{eff} \sim \frac{G_F}{\sqrt{2}} \cdot \underbrace{\frac{\alpha_{\rm EM}}{2\pi \sin^2\theta_W} \lambda_t X_t(x_t)}_{\mathcal{N}\sim 2\times 10^{-5}} \cdot (\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}$$

探测新物理的能标在 $\mathcal{N}^{-\frac{1}{2}} M_W = O(10\text{ TeV})$

过去的实验测量值是标准模型预言值的2倍

$$\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})_{\rm exp} = 1.73^{+1.15}_{-1.05} \times 10^{-10} \qquad \text{arXiv:0808.2459}$$

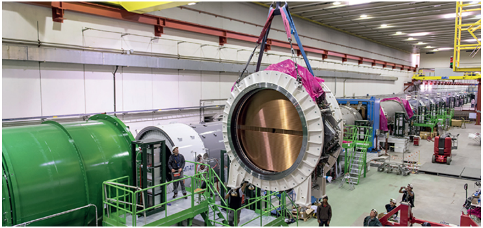
$$\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})_{\rm SM} = 9.11 \pm 0.72 \times 10^{-11} \qquad \text{arXiv:1503.02693}$$

但因为实验误差$>$ 60%，所以实验和理论还是符合的

新一代实验应运而生

新一代实验: NA62 at CERN aims at

- 2-3年内把观测到的$K^+ \to \pi^+ \nu\bar{\nu}$事例由7个提高到$O(100)$个
- 把Br($K^+ \to \pi^+ \nu\bar{\nu}$)的精度提高到10%

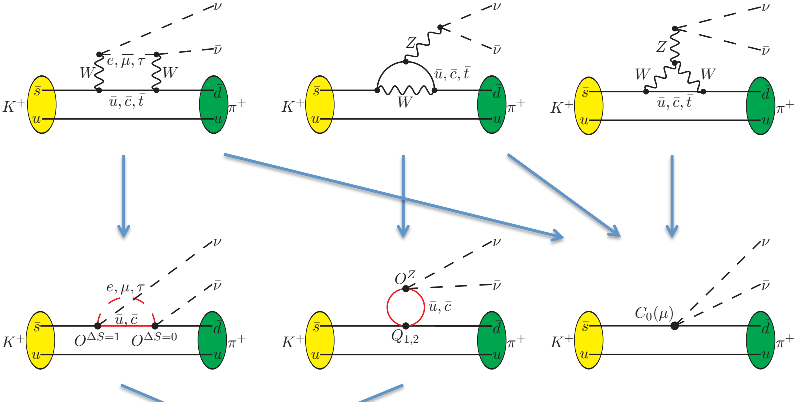


取数：2016-2018。通过2016年的数据，已经找到一个candiate event [Universe 4 (2018) 119]

$K_L \to \pi^0 \nu\bar{\nu}$

- 实验上更困难：因为初末态都是中性粒子
- only upper bound set by KEK E391a in 2010
- 新一代 J-PARC KOTO实验，就是为了寻找K_L衰变而设计运行

OPE: integrate out heavy fields Z, W, t, $\cdots$



Local SD contribution

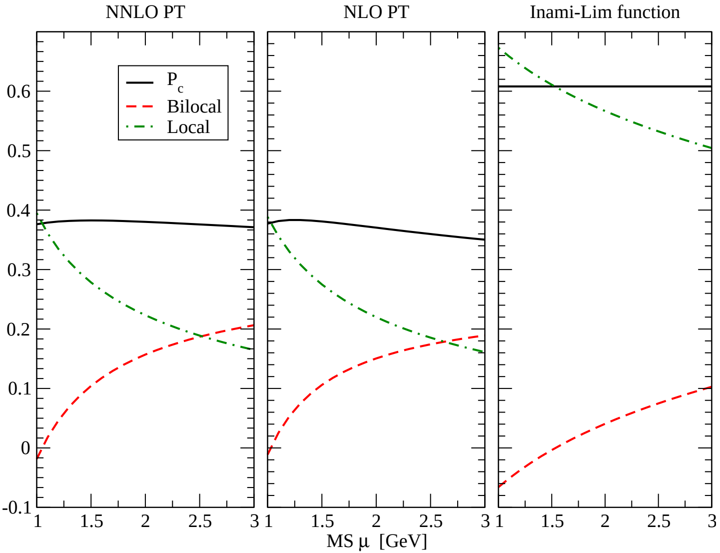
Bilocal LD contribution

Hadronic part known: $\langle \pi^+ | V_\mu | K^+ \rangle$

$\langle \pi^+ \nu \bar{\nu} | Q_A(x) Q_B(0) | K^+ \rangle$: need lattice QCD

Bilocal contribution vs local contribution

Bilocal $C_A^{\overline{\text{MS}}}(\mu)C_B^{\overline{\text{MS}}}(\mu)r_{AB}^{\overline{\text{MS}}}(\mu)$ vs Local $C_0^{\overline{\text{MS}}}(\mu)$, hep-ph/0603079



At $\mu = 2.5$ GeV, 50% charm quark 贡献来自于 bilocal 项

2阶电弱过程的强子矩阵元

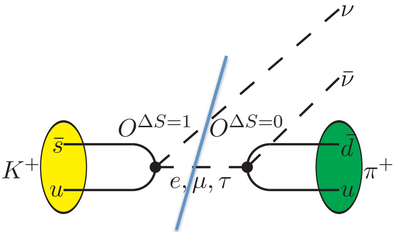
$$\int_{-T}^{T} dt \, \langle \pi^+ \nu \bar{\nu} | T \left[Q_A(t) Q_B(0) \right] | K^+ \rangle$$

$$= \sum_n \left\{ \frac{\langle \pi^+ \nu \bar{\nu} | Q_A | n \rangle \langle n | Q_B | K^+ \rangle}{M_K - E_n} + \frac{\langle \pi^+ \nu \bar{\nu} | Q_B | n \rangle \langle n | Q_A | K^+ \rangle}{M_K - E_n} \right\} \left(1 - e^{(M_K - E_n)T} \right)$$

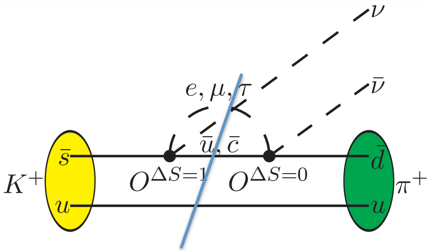
- For $E_n > M_K$, at large T, $e^{(M_K - E_n)T}$ 指数衰减
- For $E_n < M_K$, $e^{(M_K - E_n)T}$ 指数增加，必须做减除
- $\sum_n$: branch-cut主值积分被有限体积下的态求和所取代
  - 可能会导致大的有限体积修正，尤其在$E_n \to M_K$的时候

    [N. Christ, XF, G. Martinelli, C. Sachrajda, PRD91 (2015) 114510]

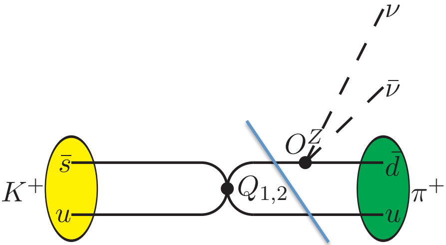
需要考虑的中间态



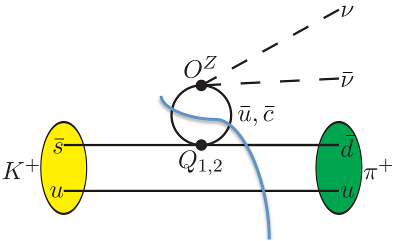
$K^+ \to \ell^+\nu$ & $\ell^+ \to \pi^+\bar{\nu}$



$K^+ \to \pi^0\ell^+\nu$ & $\pi^0\ell^+ \to \pi^+\bar{\nu}$



$K^+ \xrightarrow{H_W} \pi^+$ & $\pi^+ \xrightarrow{V_\mu} \pi^+$



$K^+ \xrightarrow{H_W} \pi^+\pi^0$ & $\pi^+\pi^0 \xrightarrow{A_\mu} \pi^+$

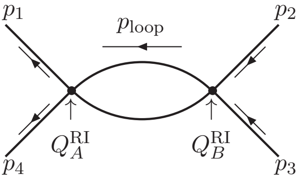
需要计算相应的2点、3点关联函数

- 计算$K_{\ell 2}$, $\pi_{\ell 2}$, $K_{\ell 3}$, pion form factor, $\langle \pi^+ | H_W | K^+ \rangle$ and $\langle \pi\pi | H_W | K \rangle$
- 暂时无需考虑3π
  - 中间态只在 $E_n < M_K$ 时贡献指数发散项以及大的有限体积修正
  - 尽管$E_{3\pi} < M_K$，但是中间态的贡献有一个相空间压低因子
- K物理 $\to$ 粲物理、B物理: 如何控制各种可能的中间态非物理效应?
  - 多道散射、多粒子态

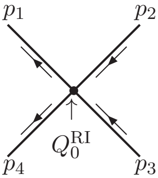
在bilocal $Q_A(x)Q_B(0)$系统中，当 $x \to 0$，SD 发散

- 引进抵消项 $X \cdot Q_0$ 来去除SD发散

$$\langle \{Q_A Q_B\}^{\rm RI} \rangle \Big|_{p_i^2=\mu_0^2} =$$



$$- X(\mu_0, a) \times$$



$$= 0$$

系数 X 可以在RI/SMOM scheme下得到

- Bilocal operator in the $\overline{\rm MS}$ scheme 可以表示成

$$\left\{ \int d^4x\, T[Q_A^{\overline{\rm MS}}(x) Q_B^{\overline{\rm MS}}(0)] \right\}^{\overline{\rm MS}}$$

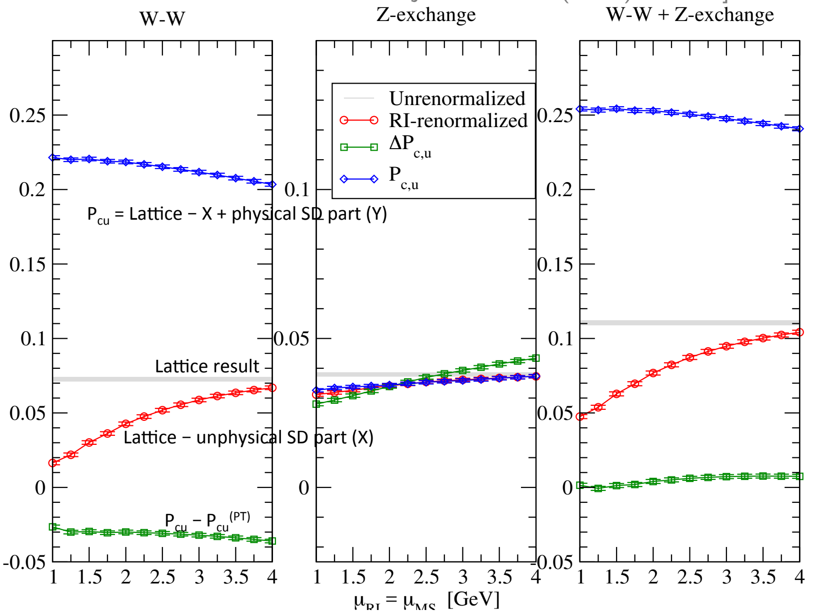
$$= Z_A Z_B \left\{ \int d^4x\, T[Q_A^{\rm lat} Q_B^{\rm lat}] \right\}^{\rm lat} + \left(-X^{\rm lat \to RI} + Y^{\rm RI \to \overline{MS}} \right) Q_0(0)$$

$X^{\rm lat \to RI}$ 可以用非微扰重整化来计算，$Y^{\rm RI \to \overline{MS}}$ 可以用微扰论来计算

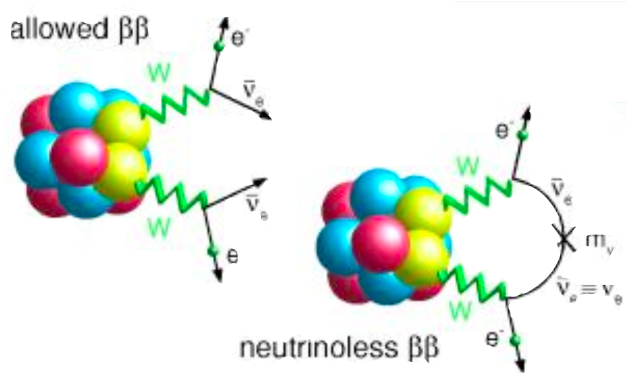
[Christ, XF, Portelli, Sachrajda, PRD93 (2016) 114517]

First results @ $m_\pi = 420$ MeV, $m_c = 860$ MeV

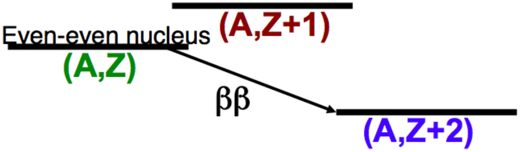
[Bai, Christ, XF, Lawson, Portelli, Schrajda, PRL118 (2017) 252001]



从味物理到核物理
——以双β衰变为例



双β衰变在自然界中确实存在



$2\nu\beta\beta$衰变是目前实验测量到的最稀有的标准模型衰变

- 双β衰变中初末态质量差ΔM很小 $\Rightarrow$ 相空间压低
- 高阶电弱相互作用 $\Rightarrow$ $\frac{\Delta M^2}{M_W^2}$压低

目前共在 ^{48}Ca(钙)、^{76}Ge(锗)、$\cdots$ ^{238}U(铀)等10种原子核中发现$2\nu\beta\beta$

- 所有的衰变，半衰期在$10^{18}-10^{21}$年 (宇宙的年龄是1.38×10^{10}年)

无中微子双β衰变

$0\nu\beta\beta$衰变是标准模型禁止的，破坏了轻子数守恒

- 一种可能的机制是Majorana中微子作为双β衰变的桥梁
- 这对于测量出中微子绝对质量提供了一种可能性

可以用成吨量级的材料，比如TeO_2来测量

CANDLES	Ca-48	60 CaF_2 crystals in liq. scint	6 kg	Construction
CARVEL	Ca-48	$^{48}CaWO_4$ crystal scint.	100 kg	
COBRA	Cd-116, Te-130	CdZnTe detectors	10 kg	R&D
CUROICINO	Te-130	TeO_2 Bolometer	11 kg	Operating
CUORE	Te-130	TeO_2 Bolometer	206 kg	Construction
DCBA	Nd-150	Nd foils & tracking chambers	20 kg	R&D
EXO200	Xe-136	Xe TPC	200 kg	Construction
EXO	Xe-136	Xe TPC	1-10t	R&D
GEM	Ge-76	Ge diodes in LN	1 t	
GERDA	Ge-76	Seg. and UnSeg. Ge in	35-40 kg	Construction

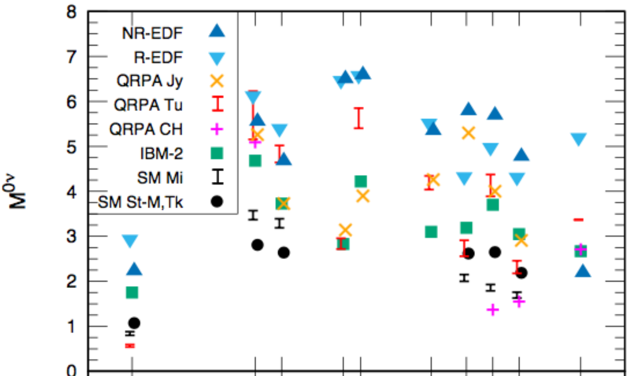
- 4个实验 (Majorana, EXO, CUORE, GERDA) 测到半衰期 $T^{0\nu}_{1/2} > 10^{25}$ yr
- 1个实验 (KamLAND-Zen) 已经测到半衰期超过1×10^{26} yr

Double β衰变：理论计算的困难

目前格点计算还主要处理轻核子

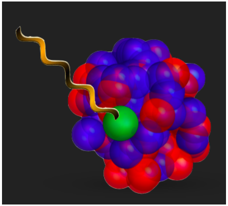
- 对于核子数为1的质子：
  - 信号：$\langle N(t)N(0)\rangle \sim e^{-M_N t}$
  - 误差：$\sim \sqrt{\langle (N(t)N(0))^2\rangle} \sim e^{-\frac{3}{2}m_\pi t}$
- 核子数为A的原子核：$\frac{\text{signal}}{\text{noise}} \sim \exp\left[-A(M_N - 3/2m_\pi)t\right] \Rightarrow$ 符号问题！

核物理模型：不同的模型给出的差异在 O(100%) 量级

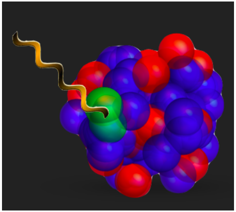


有效理论中弱流与原子核的耦合 [Detmold, talk at Lat18]

- 单体耦合占主导



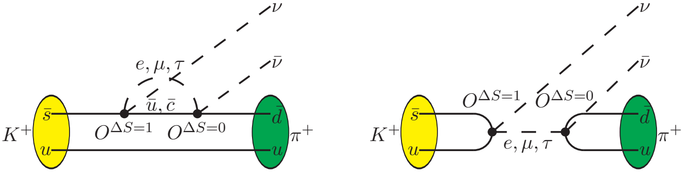
- 双核子的贡献是次要的，但却是不可忽略的



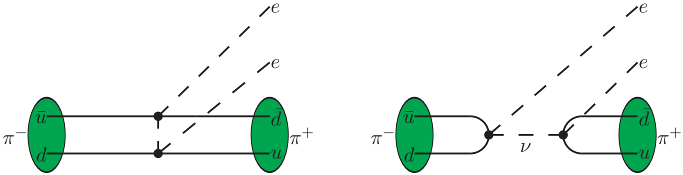
要研究多核子问题 $\Rightarrow$ 首先要了解清楚少体问题中的双β衰变

稀有K衰变与无中微子双β衰变的比较

稀有K衰变：



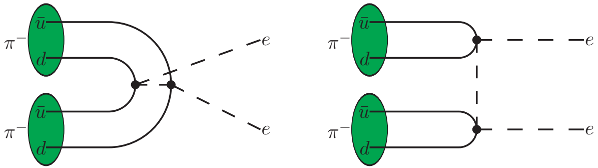
双β衰变：$\pi^-\pi^- \to ee$ 或者 $K^- \to \pi^+ ee$



从最简单的无中微子双β衰变的出发

$\pi\pi \to ee$

- 两个pion静止
- 两个电子带动量 $\vec{p}_1 = -\vec{p}_2$，$\vec{p}_{1,2} = E_{\pi\pi}/2$



- 衰变通过二阶电弱过程发生 $\pi\pi \to e_1 \bar{\nu} n \to e_1 e_2$ or $\pi\pi \to e_2 \bar{\nu} n \to e_1 e_2$
- 衰变振幅可以简写为

$$\mathcal{A} = -T_{\mathrm{lept}} \sum_n \int \frac{d^3\vec{p}_n}{(2\pi)^3} \sum_{i=1,2} \frac{\langle 0|J_{\mu L}|n\rangle\langle n|J_{\mu L}|\pi\pi\rangle}{2E_{\nu,i}E_n(E_n + E_{\nu,i} + E_i - E_{\pi\pi})}$$

with $T_{\mathrm{lept}} = 4G_F^2 V_{ud}^2 m_{\beta\beta} \bar{u}_L(p_1) u_L^c(p_2)$.

构造关联函数

$$C(t_x, t_y, t_{\pi\pi}) = \frac{1}{2!}\langle e_1 e_2|\mathcal{L}_{eff}(t_x)\,\mathcal{L}_{eff}(t_y)\,\phi_{\pi\pi}(t_{\pi\pi})|0\rangle$$

定义含时振幅 $\mathcal{M}(t)$ with $t = t_x - t_y$:

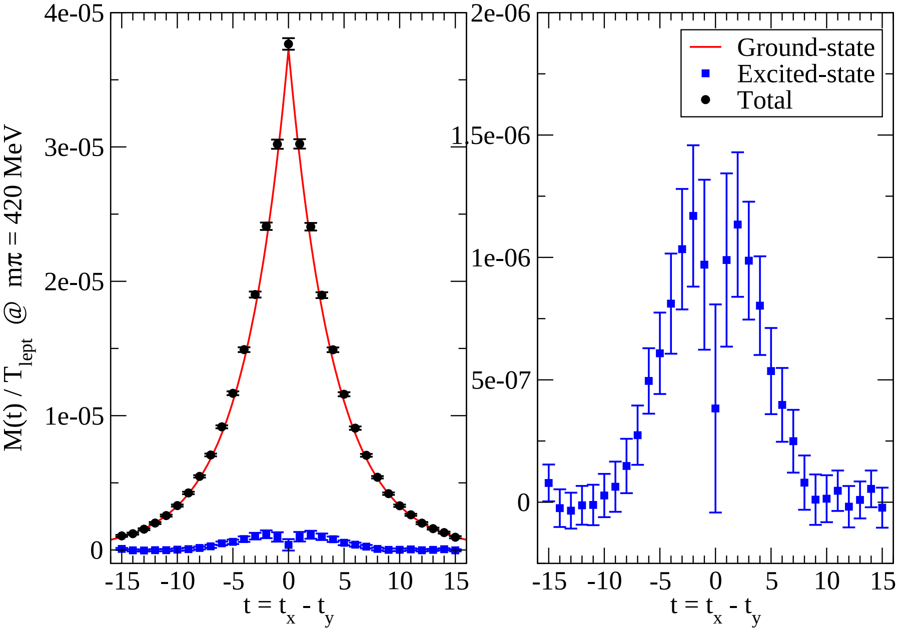
$$\mathcal{M}(t) = C(t_x, t_y, t_{\pi\pi}) / \left(V \frac{N_{\pi\pi}}{2E_{\pi\pi}} e^{E_{\pi\pi} t_{\pi\pi}}\right)$$

在大时间间隔$|t|$下，$\mathcal{M}(t)$由基态中间态主导 - $e\bar{\nu}\pi$

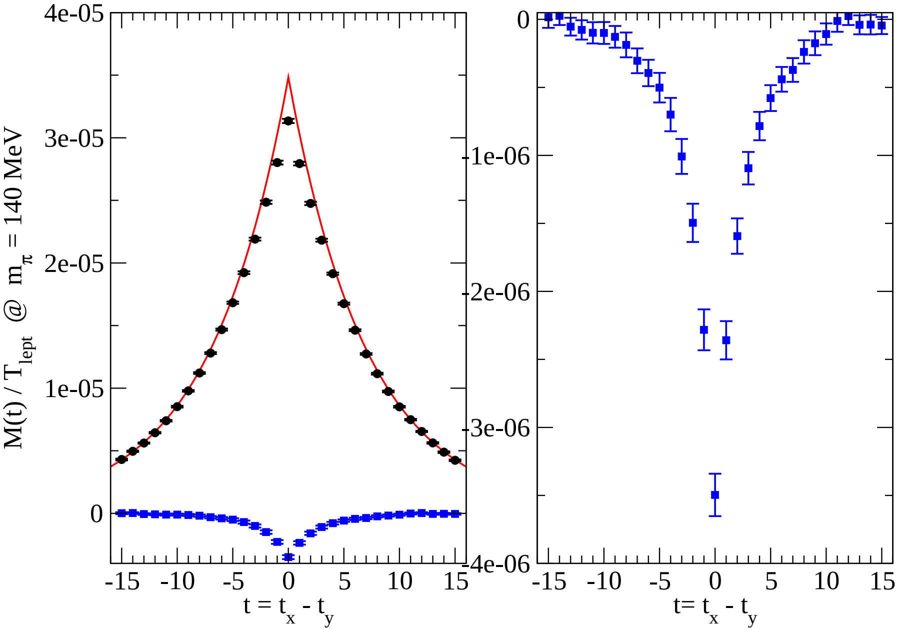
$$\mathcal{M}(t) \xrightarrow{|t|\gg 0} -T_{\mathrm{lept}} \frac{1}{V} \frac{2\langle 0|J_{\mu L}|\pi\rangle_{\mathrm{V}}\langle \pi|J_{\mu L}|\pi\pi\rangle_{\mathrm{V}}}{(2m_\pi)(2E_\nu)} e^{-m_\pi |t|}$$

对$\mathcal{M}(t)$做时间积分可得到我们想要的高阶衰变振幅

$\pi\pi \to ee$衰变振幅 @ $m_\pi = 420$ MeV



$\pi\pi \to ee$衰变振幅 @ $m_\pi = 140$ MeV



计算结果

XF, L. Jin, X. Tuo, S. Xia, PRL122 (2019) 022001

m_π [MeV]	$t_a - t_{\pi\pi}$	$\mathcal{A}^{(g)}$	$\mathcal{A}^{(e)}$	$\mathcal{A}^{(g)} + \mathcal{A}^{(e)}$
420	6	1.462(10)	0.055(13)	1.517(13)
	7		0.060(13)	1.522(13)
	8		0.052(14)	1.514(14)
140	6	1.8863(50)	−0.0664(70)	1.8199(63)
	7		−0.0660(73)	1.8203(62)
	8		−0.0665(70)	1.8199(60)

计算结果统计误差$< 1\%$

$$\left.\frac{\mathcal{A}(\pi\pi \to ee)}{F_\pi^2\, T_{\mathrm{lept}}}\right|_{m_\pi = 420\ \mathrm{MeV}} = 1.517(13),$$

$$\left.\frac{\mathcal{A}(\pi\pi \to ee)}{F_\pi^2\, T_{\mathrm{lept}}}\right|_{m_\pi = 140\ \mathrm{MeV}} = 1.820(6).$$

在 $m_\pi = 420$ and 140 MeV, $\mathcal{A}(\pi\pi \to ee)$ 比领头阶手征微扰论小24% 和 9%

$$\frac{\mathcal{A}^{\mathrm{LO}}(\pi\pi \to ee)}{F_\pi^2\, T_{\mathrm{lept}}} = 2$$

展望

格点QCD发展到今天，即将进入Exaflop 时代（每秒计算10^{18}次）

- 常规物理量， 期待精度大幅提高
- 非常规物理量，像长程相互作用过程，很值得我们进一步去探索
- 在味物理方面，我们希望能为高精度前沿领域寻找新物理提供相关的低能QCD的信息
- 在味物理中发展出来的方法也被用到核物理的计算中去
    - 可以帮助探索与核物质相关的稀有过程
    - 也许有朝一日，核物理会成为新的味物理

Backup slides

Neutrinos: Dirac or Majorana?

Start from Dirac fermion

- Lagrangian density for a classical Dirac field

$$\mathcal{L}_{\mathrm{Dirac}} = \bar{\psi}(i\gamma^0\partial_t + i\vec{\gamma}\cdot\vec{\nabla} - m)\psi$$

- Using Weyl representation of γ, one can write ψ as

$$\psi = \begin{pmatrix}\phi\\ \chi\end{pmatrix}, \quad \psi_L = \begin{pmatrix}0\\ \chi\end{pmatrix}, \quad \psi_R = \begin{pmatrix}\phi\\ 0\end{pmatrix}$$

- Lagrangian becomes

$$\mathcal{L}_{\mathrm{Dirac}} = \chi^\dagger \underbrace{i(\partial_t - \vec{\sigma}\cdot\vec{\nabla})}_{O_-}\chi + \phi^\dagger \underbrace{i(\partial_t + \vec{\sigma}\cdot\vec{\nabla})}_{O_+}\phi - m\underbrace{(\chi^\dagger\phi + \phi^\dagger\chi)}_{\text{Dirac mass}}$$

- Lorentz transform for the fermionic fields $\psi(x) \to \psi'(x) = \Lambda_{\frac{1}{2}}\psi(\Lambda^{-1}x)$

$$\Lambda_{\frac{1}{2}} = \exp\left(-\frac{i}{2}\omega_{\mu\nu}S^{\mu\nu}\right), \quad \omega_{ij} = \epsilon_{ijk}\theta_k, \quad \omega_{0i} = \beta_i, \quad S^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu]$$

Under infinitesimal rotations $\vec{\theta}$ and boosts $\vec{\beta}$, χ and ϕ transform differently

$$\chi'(x) = (1 - i\vec{\theta}\cdot\frac{\vec{\sigma}}{2} + \vec{\beta}\cdot\frac{\vec{\sigma}}{2})\chi(\Lambda^{-1}x)$$

$$\phi'(x) = (1 - i\vec{\theta}\cdot\frac{\vec{\sigma}}{2} - \vec{\beta}\cdot\frac{\vec{\sigma}}{2})\phi(\Lambda^{-1}x)$$

- Realizing that $\tilde{\phi} = i\sigma_2\phi^*$ behaves like a left-handed spinor, one can define

$$\text{Left hand: } \eta = \frac{\chi + \tilde{\phi}}{\sqrt{2}}, \quad \text{Right hand: } \xi = -i\frac{\tilde{\chi} + \phi}{\sqrt{2}}$$

- Define the charge-conjugate field

$$\psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix}, \quad \psi^c = C\bar{\psi}^T = i\gamma^2\psi^* = \begin{pmatrix} -\tilde{\chi} \\ \tilde{\phi} \end{pmatrix}$$

One can construct self-charge-conjugate (Majorana) field N_1 and N_2

$$N_1 = \frac{\psi + \psi^c}{\sqrt{2}}, \quad N_2 = -i\frac{\psi - \psi^c}{\sqrt{2}}$$

N_1 is purely related to η and N_2 is related to ξ

$$N_1 = N_{1L} + N_{1L}^c = \begin{pmatrix} 0 \\ \eta \end{pmatrix} + \begin{pmatrix} -\tilde{\eta} \\ 0 \end{pmatrix}, \quad N_2 = N_{2L} + N_{2L}^c = \begin{pmatrix} 0 \\ \tilde{\xi} \end{pmatrix} + \begin{pmatrix} \xi \\ 0 \end{pmatrix} = (N_{2R})$$

- Lagrangian density for Dirac field can also be expressed as

$$\mathcal{L}_{\rm Dirac} = \mathcal{L}_{\rm Majorana}(N_1) + \mathcal{L}_{\rm Majorana}(N_2)$$

$$\begin{aligned}
\mathcal{L}_{\rm Majorana}(N_1) &= \frac{1}{2}\overline{N_1}(i\gamma^\mu\partial_\mu - m)N_1 \\
&= \overline{N_{1L}}(i\gamma^\mu\partial_\mu)N_{1L} - \frac{1}{2}m\left(\overline{N_{1L}}(N_{1L})^c + \text{h.c.}\right) \\
&= \underbrace{\eta^\dagger O_-\eta}_{\text{Left handed}} \quad -\frac{1}{2}m\underbrace{\left(\eta^\dagger(-\tilde{\eta}) + \text{h.c.}\right)}_{\text{Majorana mass}}
\end{aligned}$$

  - Dirac fermion consists of a pair of mass degenerate Majorana fermion
  - Majorana fermion also satisfies Dirac equation

- Under global phase transformation

$$\begin{aligned}
\text{Dirac: } \chi \to e^{i\alpha}\chi, \quad \phi \to e^{i\alpha}\phi \quad &\Rightarrow \quad \text{lepton number conservation} \\
\text{Majorana: } \eta \to e^{i\alpha}\eta, \quad \tilde{\eta} \to e^{-i\alpha}\tilde{\eta} \quad &\Rightarrow \quad \text{lepton number violation}
\end{aligned}$$

  - Electric charge conservation forces charged fermion to be Dirac type
  - Neutrino can be Dirac, Majorana or the mixed type

Light-neutrino exchange in $0\nu2\beta$ decay

Minimal extension of SM – exchange of three light Majorana neutrinos

- Effective Lagrangian for β decay

$$\mathcal{L}_{eff} = 2\sqrt{2}G_F V_{ud}(\overline{u}_L\gamma_\mu d_L)(\overline{e}_L\gamma_\mu\nu_{eL})$$

- Effective Hamlitonian for 2β decay

$$\mathcal{H}^{2\beta}_{eff} = \frac{1}{2!}\int d^4x\, \mathcal{L}_{eff}(x)\mathcal{L}_{eff}(0)$$

- Neutrino flavor eigenstate mixes with three mass eigenstates

$$\overline{e}_L\gamma_\mu\nu_{eL} \quad\rightarrow\quad \sum_k \overline{e}_L\gamma_\mu U_{ek}\nu_{kL}$$

U_{ek} is the mixing matrix element.

- These neutrinos are very light

Long-distance contribution dominated

Light-neutrino exchange in $0\nu2\beta$ decay

Assume that $0\nu2\beta$ is mediated by exchange of light Majorana neutrinos

$$
\begin{aligned}
& \sum_k \overline{e}_L(x)\gamma_\mu U_{ek}\underbrace{\nu_{kL}(x)\,\overline{e}_L(0)\gamma_\nu U_{ek}\nu_{kL}}(0) \\
= \quad & -\sum_k \overline{e}_L(x)\gamma_\mu U_{ek}\underbrace{\nu_{kL}(x)\,\overline{\nu^c_{k}}_L}(0)\gamma_\nu U_{ek} e^c_L(0) \\
= \quad & -\sum_k \overline{e}_L(x)\gamma_\mu U_{ek} P_L \left(\int \frac{d^4q}{(2\pi)^4} \frac{-i\not{q} + m_k}{q^2 + m_k^2} e^{iqx} \right) P_L \gamma_\nu U_{ek} e^c_L(0) \\
\approx \quad & -m_{\beta\beta} \int \frac{d^4q}{(2\pi)^4} \frac{e^{iqx}}{q^2} \overline{e}_L(x)\gamma_\mu\gamma_\nu e^c_L(0)
\end{aligned}
$$

In the last step, $\not{q}$ vanishes and m_k enters into the effective mass $m_{\beta\beta}$

$$
m_{\beta\beta} = \sum_k m_k U_{ek}^2
$$

$0\nu2\beta$ decay amplitude is proportional to the absolute neutrino mass