暗物质和真空稳定性

毕效军
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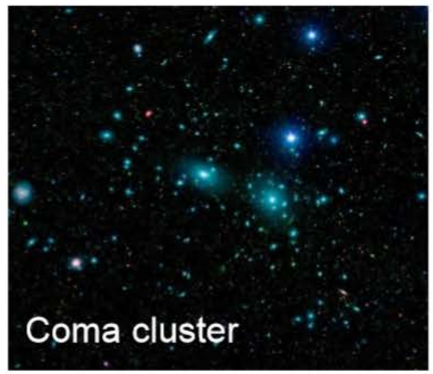
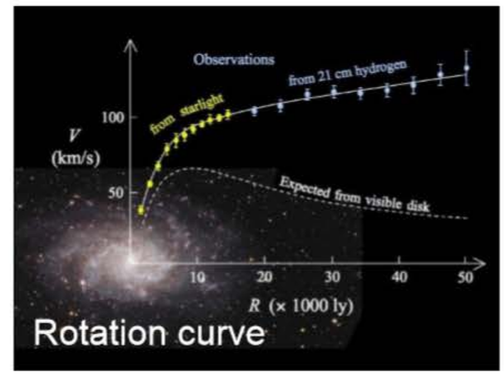
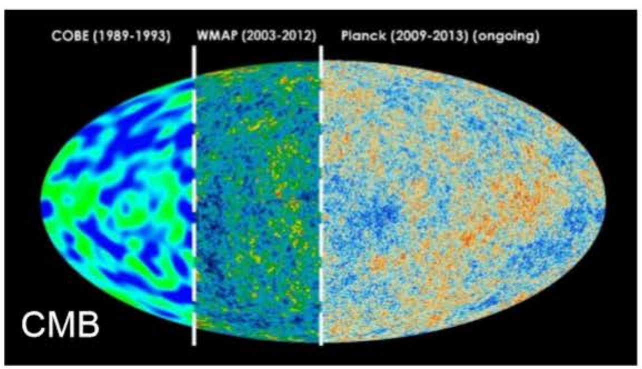
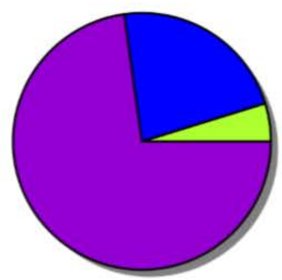
暗物质问题

The astrophysical and cosmological observations have provided compelling evidences of the existence of **dark matter (DM)**.









Planck 2015
[1502.01589]

Cold DM (25.8%)

$\Omega_c h^2 = 0.1186 \pm 0.0020$

Baryons (4.8%)

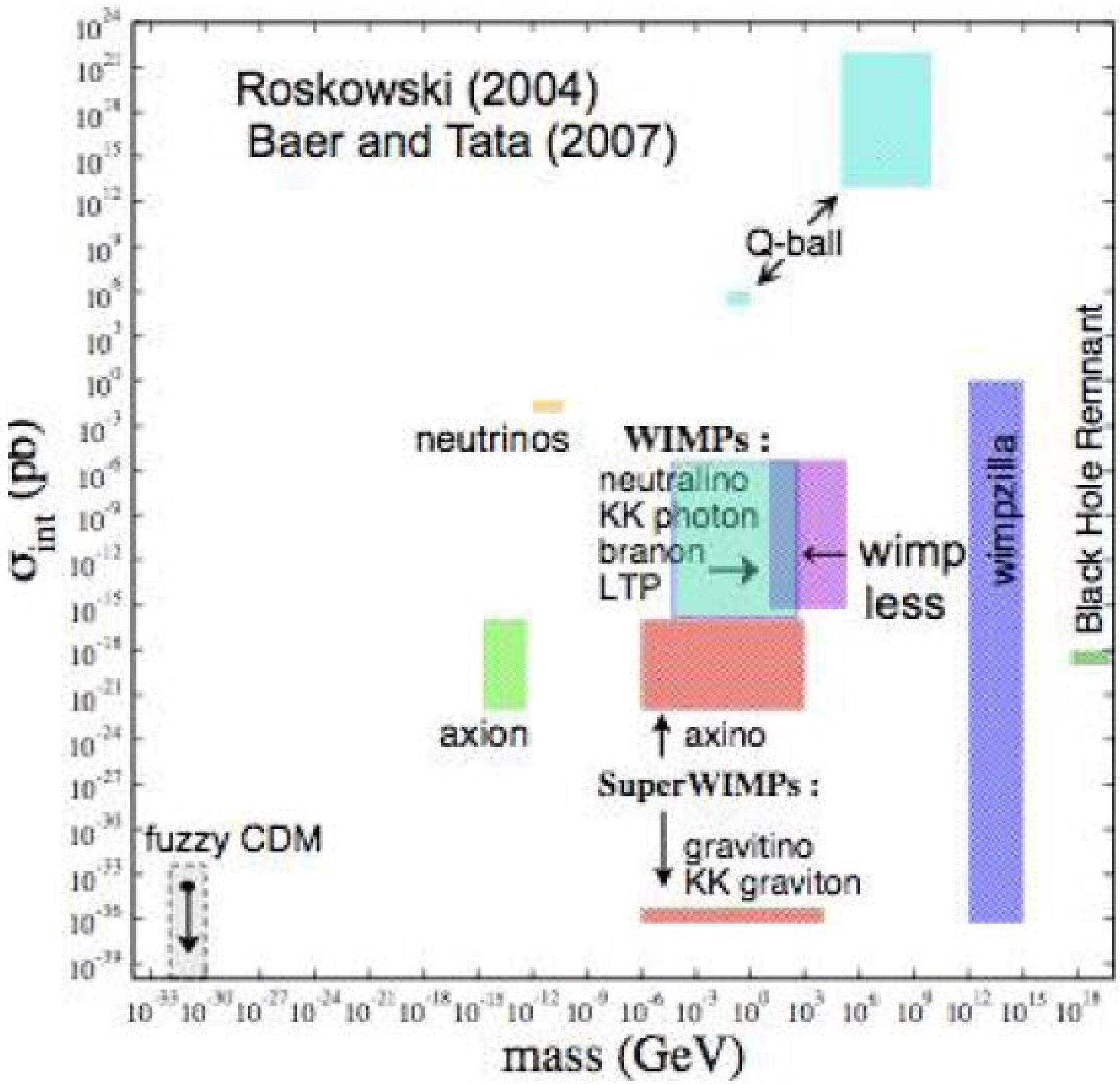
$\Omega_b h^2 = 0.02226 \pm 0.00023$

Dark energy (69.3%)

$\Omega_\Lambda = 0.692 \pm 0.012$

Candidates of the cold dark matter — stable、neutral、weak interacting

- There are dozen
- Weakly Interactin
relics of Big Bang i
independently pro
- such as neutralin



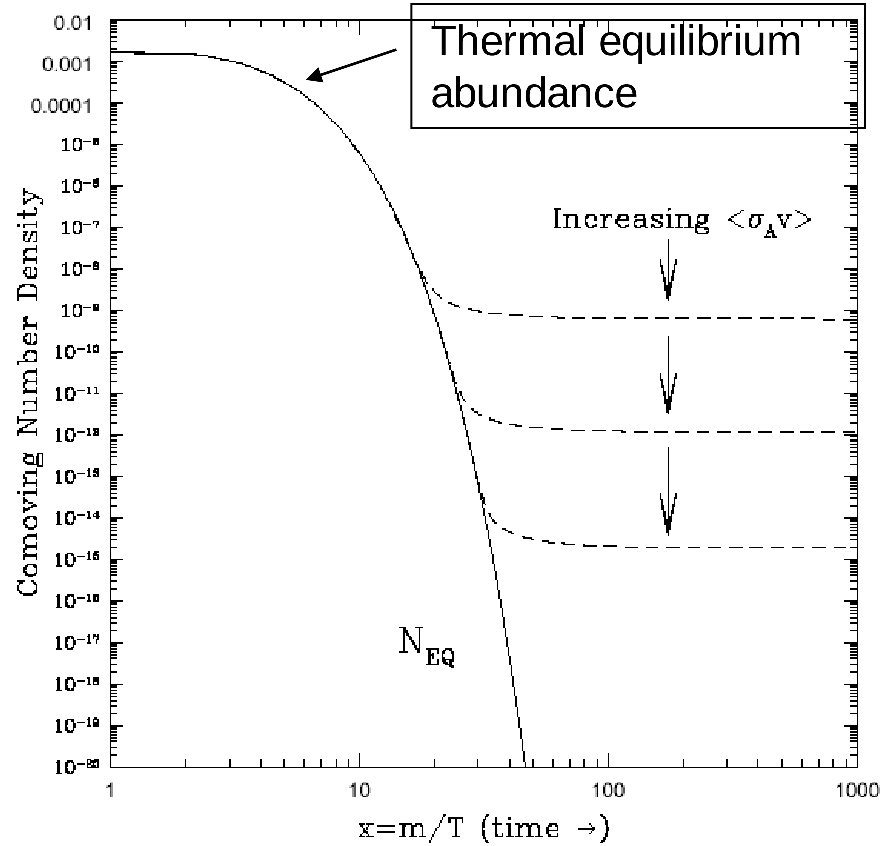
The WIMP miracle: for typical gauge couplings and masses of order the electroweak scale, $\Omega_{wimp}h^2 \approx 0.1$ (within factor of 10 or so)

Thermal history of the WIMP (thermal production)

At T >> m, $f + \bar{f} \leftrightarrow \chi + \chi$

At T < m, $\chi + \chi \rightarrow f + \bar{f}$

At T ~ m/22, $\Gamma = n\langle \sigma v \rangle \sim H$, decoupled, relic density is inversely proportional to the interaction strength $\Omega_\chi h^2 \approx \frac{3\cdot 10^{-27} cm^3 s^{-1}}{\langle \sigma v \rangle_{T_f}}$

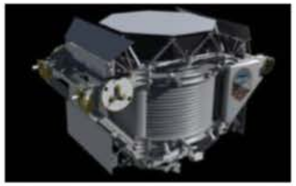


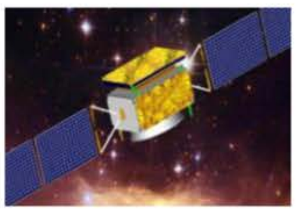
For the weak scale interaction and mass scale (non-relativistic dark matter particles) $\langle \sigma v \rangle \sim 3\cdot 10^{-26} cm^3 s^{-1}$,
if $\alpha \sim 10^{-2}$ $M_{\text{weak}} \sim 100 GeV$ and $v^2 \approx c^2 / 22$

WIMP is a natural dark matter candidate giving correct relic density (proposed trying to solve hierarchy problem).

WIMP models

Phys. Rev. D97 (2018) 035021, arXiv:1711.05622

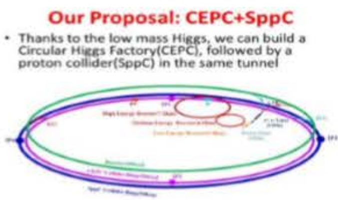




Inirect detection

WIMPS

Collider detection



Direct detection

LUX

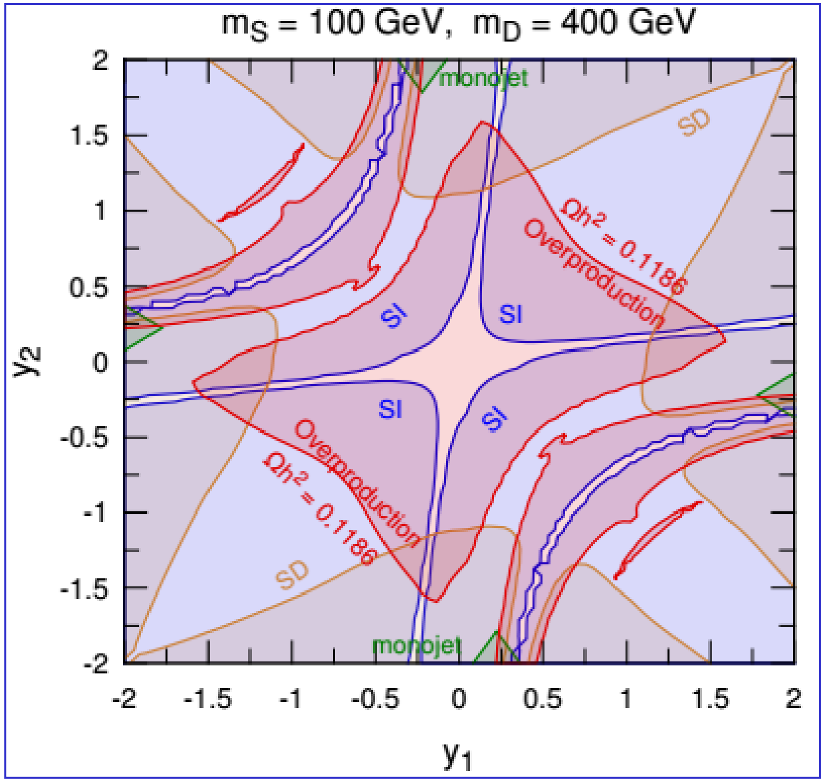
WIMP – electroweak multiplets

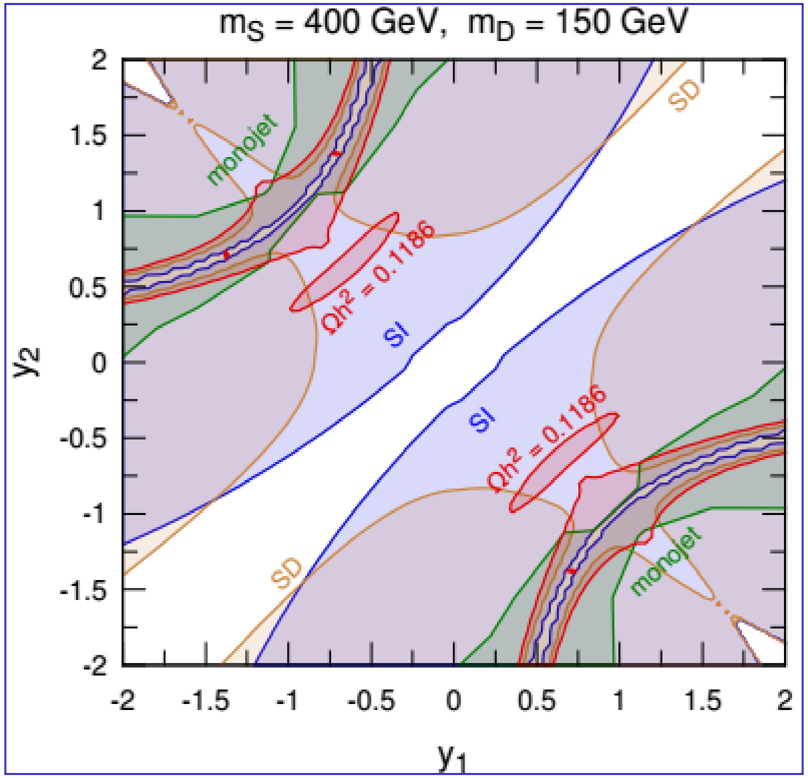
- SUSY neutralino 1-2-3

For DM phenomenology, it is quite natural to construct WIMP models by extending the SM with a dark sector consisting of $SU(2)_L$ multiplet, whose neutral components could provide a viable DM candidate

- 1 multiplet in a high dim rep. : **minimal DM model** [Cirelli et al., 0512090]
    - (DM stability is explained by an 'accidental symmetry')
- 2 types of multiplets: an artificial Z_2 symmetry is usually needed
    - **Singlet-doublet DM model** [0510064, 0705.4493, 1109,2604]
    - **Doublet-triplet DM model** [1403.7744, 1707.03094]
    - **Triplet-quadruplet DM model** [1601.01354, **1711.05622**]

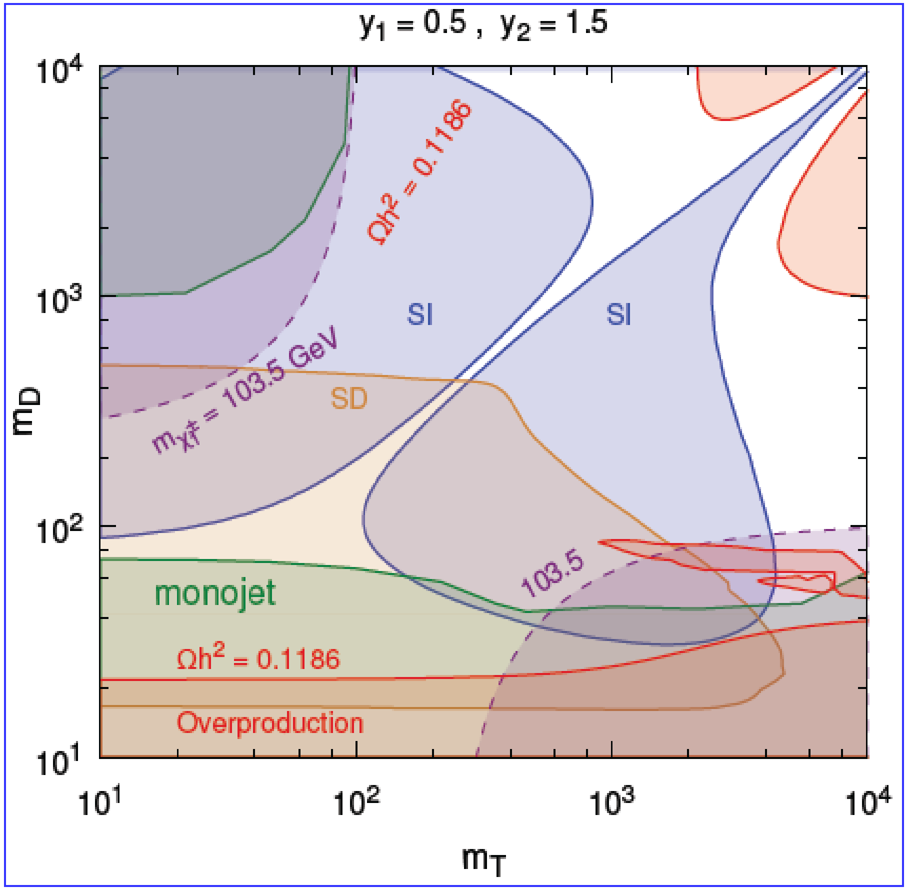
Constraints from DM searches (SDFDM)

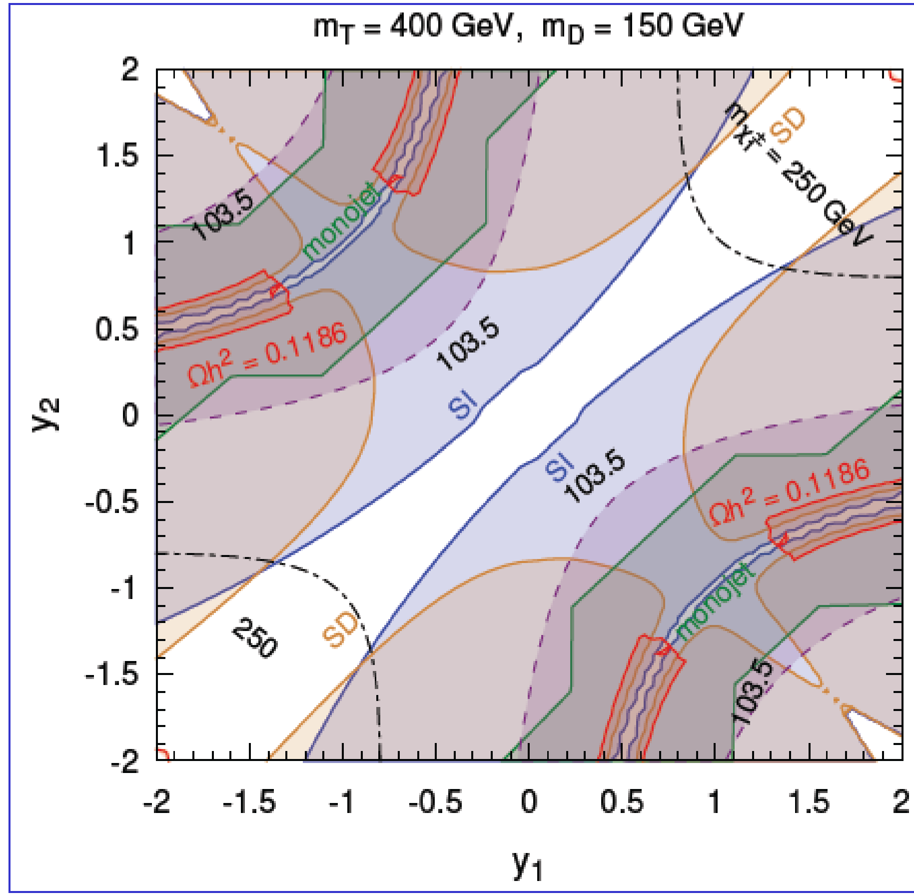




- ***Parameter space is stringently constrained by direct detection, except for some regions where the DM couplings to Z and Higgs are suppressed***

Constraints from DM searches (DTFDM)



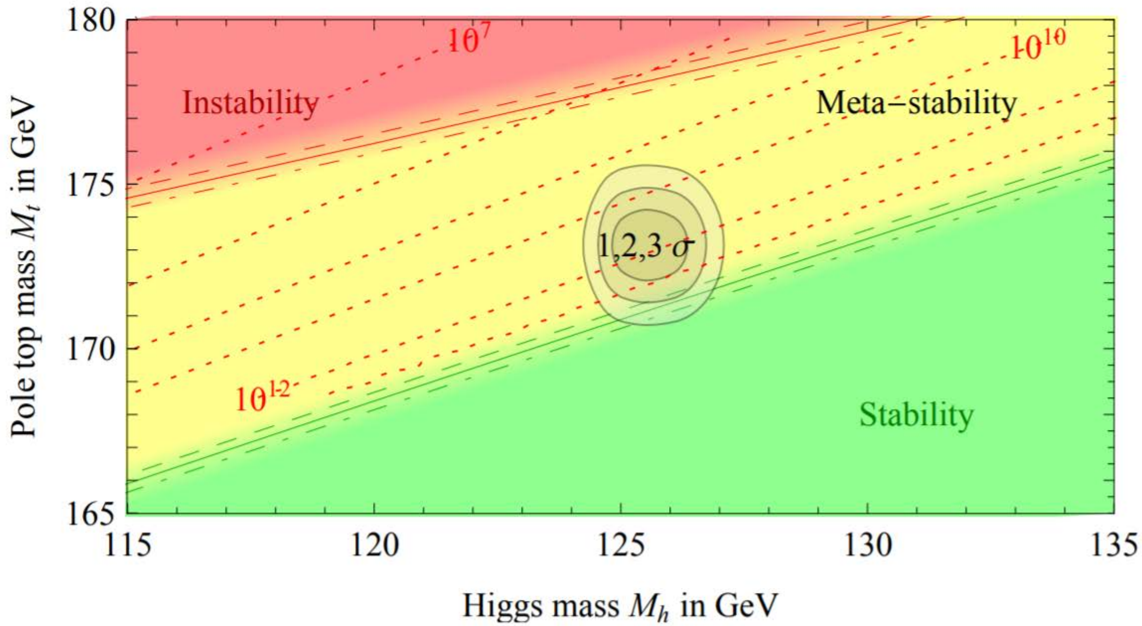


Details are given by ZhaoHuan's talk.

Higgs vacuum stability

- The SM Higgs mass implies that the vacuum is metastable.

Degrassi et al, JHEP1208, 098 (2012)



- How does dark sector affect the vacuum stability?

Matching and running

To extrapolate a theory from the electroweak scale to high energies, we require two ingredients:

(1) the initial values of couplings at low energy scale;

(2) the RGEs of all the running parameters.

There are two different matching schemes:

Tree level matching: $\begin{cases} \beta_{SM} & for\ \ \Lambda < Q \\ \beta_{SM} + \beta_{BSM} & for\ \ \Lambda > Q \end{cases}$ with continuous values.

Loop level matching: consider the loop corrections of BSM at Q scale, and then we use the complete RGE.

We would like to compare these two different schemes.

Initial values in Msbar scheme

So we shall always work in MS bar scheme and determine the MS bar parameters in terms of physical observables:

Observables <=> OS parameters <=> MS bar parameters

This can be easily done by using the following relations:

$$\theta_0 = \theta_{\rm OS} - \delta\theta_{\rm OS} = \theta(\bar{\mu}) - \delta\theta_{\overline{\rm MS}} \quad \text{or} \quad \theta(\bar{\mu}) = \theta_{\rm OS} - \delta\theta_{\rm OS} + \delta\theta_{\overline{\rm MS}}$$

The divergent parts are canceled with each other, while for the finite part there is a higher-order discrapancy, but this can be ignored for one-loop level.

$$\theta(\bar{\mu}) = \theta_{\rm OS} - \delta\theta_{\rm OS}|_{\rm fin} + \Delta_\theta$$

Strumia et .al [arXiv:1307.3536]

Inputs and tree level relation

Input values of the SM observables:

$$
\begin{array}{rcll}
M_W & = & 80.384 \pm 0.014 \text{ GeV} & \text{Pole mass of the } W \text{ boson} \\
M_Z & = & 91.1876 \pm 0.0021 \text{ GeV} & \text{Pole mass of the } Z \text{ boson} \\
M_h & = & 125.15 \pm 0.24 \text{ GeV} & \text{Pole mass of the higgs} \\
M_t & = & 173.34 \pm 0.76 \pm 0.3 \text{ GeV} & \text{Pole mass of the top quark} \\
V \equiv (\sqrt{2}G_\mu)^{-1/2} & = & 246.21971 \pm 0.00006 \text{ GeV} & \text{Fermi constant for } \mu \text{ decay} \\
\alpha_3(M_Z) & = & 0.1184 \pm 0.0007 & \overline{\text{MS}} \text{ gauge SU}(3)_c \text{ coupling (5 flavours)}
\end{array}
$$

the SM fundamental parameters ($\lambda, m, y_t, g_2, g_Y$) can be defined by those observables:

$$
V_0 = -\frac{m_0^2}{2}|H_0|^2 + \lambda_0 |H_0|^4, \qquad \lambda_{\text{OS}} = \frac{G_\mu}{\sqrt{2}} M_h^2, \qquad m_{\text{OS}}^2 = M_h^2.
$$

$$
y_{t\text{OS}} = 2\left(\frac{G_\mu}{\sqrt{2}} M_t^2\right)^{1/2}, \quad g_{2\text{OS}} = 2\left(\sqrt{2}\,G_\mu\right)^{1/2} M_W, \quad g_{Y\text{OS}} = 2\left(\sqrt{2}\,G_\mu\right)^{1/2} \sqrt{M_Z^2 - M_W^2}\ .
$$

One loop level corrections

$$\delta^{(1)}\lambda_{\rm OS} = \frac{G_\mu}{\sqrt{2}} M_h^2 \left\{ \Delta r_0^{(1)} + \frac{1}{M_h^2}\left[\frac{T^{(1)}}{v_{\rm OS}} + \delta^{(1)} M_h^2\right]\right\} \qquad \delta^{(1)} m_{\rm OS}^2 = 3\frac{T^{(1)}}{v_{\rm OS}} + \delta^{(1)} M_h^2$$

$$\delta^{(1)} y_{t_{\rm OS}} = 2\left(\frac{G_\mu}{\sqrt{2}} M_t^2\right)^{1/2}\left(\frac{\delta^{(1)} M_t}{M_t} + \frac{\Delta r_0^{(1)}}{2}\right) \qquad \delta^{(1)} g_{2_{\rm OS}} = \left(\sqrt{2}\, G_\mu\right)^{1/2} M_W \left(\frac{\delta^{(1)} M_W^2}{M_W^2} + \Delta r_0^{(1)}\right)$$

$$\delta^{(1)} g_{Y_{\rm OS}} = \left(\sqrt{2}\, G_\mu\right)^{1/2} \sqrt{M_Z^2 - M_W^2}\left(\frac{\delta^{(1)} M_Z^2 - \delta^{(1)} M_W^2}{M_Z^2 - M_W^2} + \Delta r_0^{(1)}\right)$$

these corrections only depend on mass corrections of higgs and gauge boson and Δr_0

we need to calculate the loop corrections to the muon decay process:

$$\frac{G_\mu}{\sqrt{2}} = \frac{1}{2v_0^2}(1 + \Delta r_0) \qquad \Delta r_0^{(1)} = V_W^{(1)} - \frac{A_{WW}^{(1)}}{M_W^2} + \frac{\sqrt{2}}{G_\mu}\mathcal{B}_W^{(1)} + \mathcal{E}^{(1)}$$

V_W is the vertex contribution; A_{WW} is the W self-energy at zero momentum;

$\mathcal{B}_W$ is the box contribution; $\mathcal{E}$ is the term due to renormalization of exernal legs;

A. Sirlin and R. Zucchini, Nucl. Phys. B 266 (1986) 389

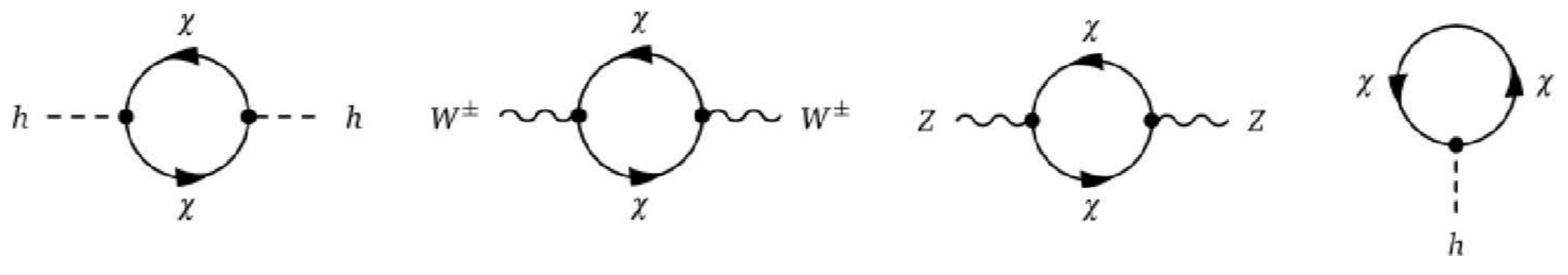
Msbar parameters

- Adopting the relation $\theta(\bar{\mu}) = \theta_{\rm OS} - \delta\theta_{\rm OS}|_{\rm fin}$ we have

$$\lambda_{\overline{\rm MS}} = \frac{G_\mu}{\sqrt{2}} M_h^2 - \delta^{(1)}\lambda_{\rm OS}|_{\rm fin}, \quad m^2_{\overline{\rm MS}} = M_h^2 - \delta^{(1)} m^2_{\rm OS}|_{\rm fin}, \quad y_{t,\overline{\rm MS}} = 2\left(\frac{G_\mu}{\sqrt{2}} M_t^2\right)^{1/2} - \delta^{(1)} y_{t,{\rm OS}}|_{\rm fin}, \tag{17}$$

$$g_{2,\overline{\rm MS}} = 2\left(\sqrt{2}\,G_\mu\right)^{1/2} M_W - \delta^{(1)} g_{2,{\rm OS}}|_{\rm fin}, \quad g_{Y,\overline{\rm MS}} = 2\left(\sqrt{2}\,G_\mu\right)^{1/2} \sqrt{M_Z^2 - M_W^2} - \delta^{(1)} g_{Y,{\rm OS}}|_{\rm fin}.$$

these corrections only depend on mass corrections of higgs and gauge boson and Δr_0



Singlet-doublet DM model

Dark sector Weyl fermions ($SU(2)_L \times U(1)_Y$):

$$S \in (\mathbf{1}, 0), \quad D_1 = \begin{pmatrix} D_1^0 \\ D_1^- \end{pmatrix} \in \left(\mathbf{2}, -\frac{1}{2}\right), \quad D_2 = \begin{pmatrix} D_2^+ \\ D_2^0 \end{pmatrix} \in \left(\mathbf{2}, \frac{1}{2}\right)$$

Gauge invariant kinetic terms, mass terms and Yukawa couplings:

$$\mathcal{L}_S = iS^\dagger \bar{\sigma}^\mu D_\mu S - \frac{1}{2}(m_S SS + \text{h.c.})$$

$$\mathcal{L}_D = iD_1^\dagger \bar{\sigma}^\mu D_\mu D_1 + iD_2^\dagger \bar{\sigma}^\mu D_\mu D_2 - (m_D b_{ij} D_1^i D_2^j + \text{h.c.})$$

$$\mathcal{L}_{HSD} = y_1 c_{ij} S D_1^i H^j + y_2 d_{ij} S D_2^i \widetilde{H}^j + \text{h.c.}$$

b_{ij}, c_{ij}, and d_{ij} can be decoded from CG coefficients multiplied by a normalizing factor

There are four independent parameters: m_S, m_D, y_1, y_2

modify the vacuum stability

Beta function

First of all, we would like to calculate the β function of g_1, g_2, g_3;

(1) $\beta(g_1)=\beta^{SM}(g_1)+\frac{g_1^3}{(4\pi)^2}\left(\sum_i \frac{4}{3}n_f Q_i^2\right)=\beta^{SM}(g_1)+\frac{g_1^3}{(4\pi)^2}\left(\frac{4}{3}\times 2\times\left(-\frac{1}{2}\right)^2+\frac{4}{3}\times 2\times\left(\frac{1}{2}\right)^2\right)\times\frac{1}{2}=\beta^{SM}(g_1)+\frac{g_1^3}{(4\pi)^2}\frac{2}{3}=\frac{1}{16\pi^2}\left(\frac{41}{6}+\frac{2}{3}\right)g_1^3$

(2) $\beta(g_2)=\beta^{SM}(g_2)+\left(\frac{-g_2^3}{(4\pi)^2}\left(-\sum_i \frac{4}{3}n_f C(r)\right)\right)\xrightarrow[C(2)=tr[t^3t^3]=\frac{1}{2}]{C(1)=tr[t^3t^3]=0}\beta^{SM}(g_2)+\frac{g_2^3}{(4\pi)^2}\left(\frac{4}{3}\times\frac{1}{2}+\frac{4}{3}\times\frac{1}{2}\right)\times\frac{1}{2}=\frac{1}{16\pi^2}\left(-\frac{19}{6}+\frac{2}{3}\right)g_2^3;$

(3) $\beta(g_3)=\beta^{SM}(g_3)=\frac{1}{16\pi^2}(-7)g_3^3;$

For other Yukawa couplings we use the formulas:

$$\frac{\partial g}{\partial \ln\mu}=\beta(g) \qquad \beta(g)=M\frac{\partial}{\partial M}\left(-\delta_g+\frac{1}{2}g\sum_i \delta_{Z_i}\right)$$

For higgs quartic couplings we use effective potential:

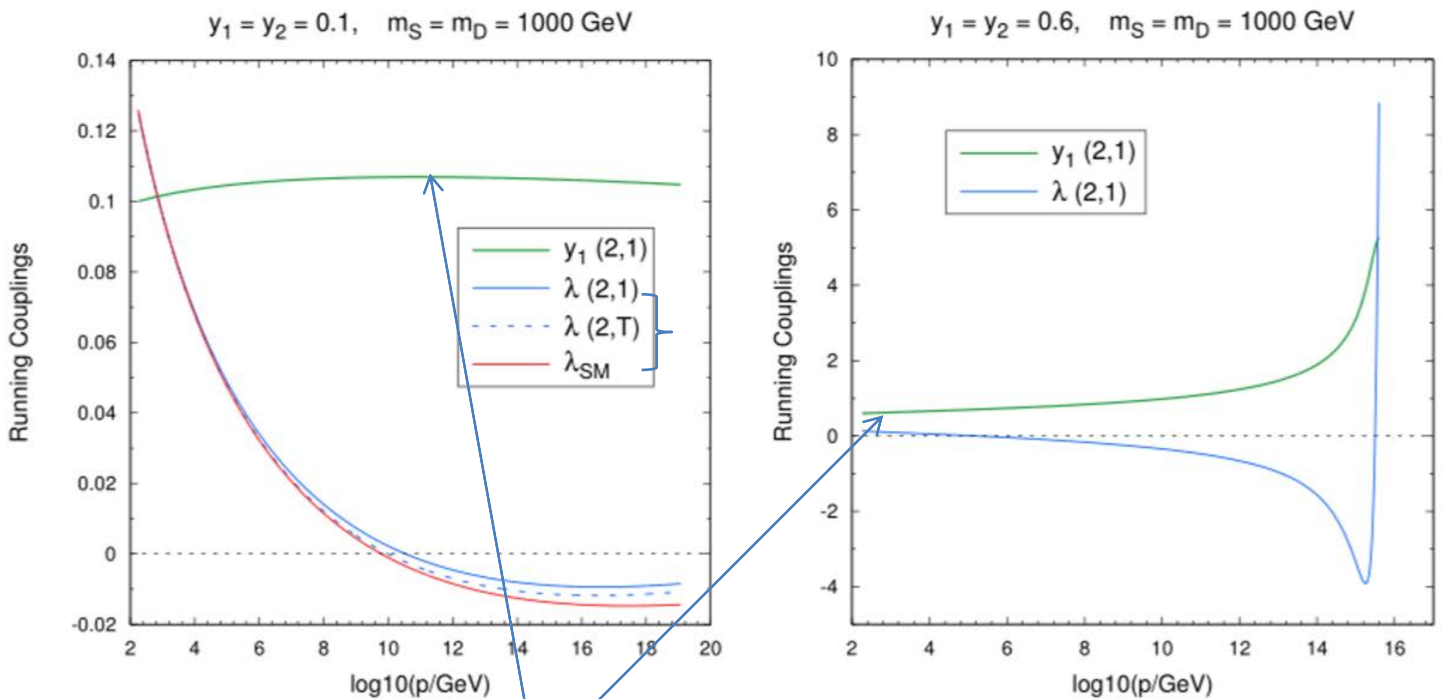
$$\left(\frac{1}{4}\beta(\lambda)-\gamma\lambda\right)\phi_c^4=-\mu_R\frac{\partial V_{1-2}}{\partial\mu_R} \qquad \gamma=\frac{1}{2}M\frac{\partial}{\partial M}\delta_h$$

Use PyR@TE 2.0 compute the two-loop RGEs

For 1-2 model: $V_{eff,1-2}=-\frac{2}{64\pi^2}m_-^4(\phi_c)\left[\ln\frac{m_-^2(\phi_c)}{\mu_R^2}-\frac{3}{2}\right]-\frac{2}{64\pi^2}m_+^4(\phi_c)\left[\ln\frac{m_+^2(\phi_c)}{\mu_R^2}-\frac{3}{2}\right]$

$$m_-=\frac{(M_S+M_D)-\sqrt{(M_S+M_D)^2-4\left[M_SM_D-\phi_c^2\left(A^2y_1^2+B^2y_2^2\right)\right]}}{2},\quad m_+=\frac{(M_S+M_D)+\sqrt{(M_S+M_D)^2-4\left[M_SM_D-\phi_c^2\left(A^2y_1^2+B^2y_2^2\right)\right]}}{2}$$

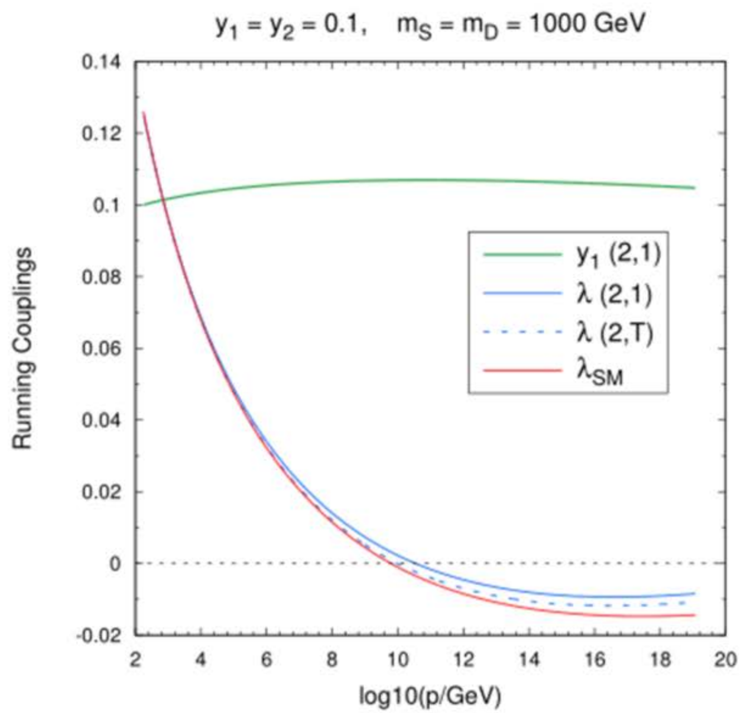
Running couplings

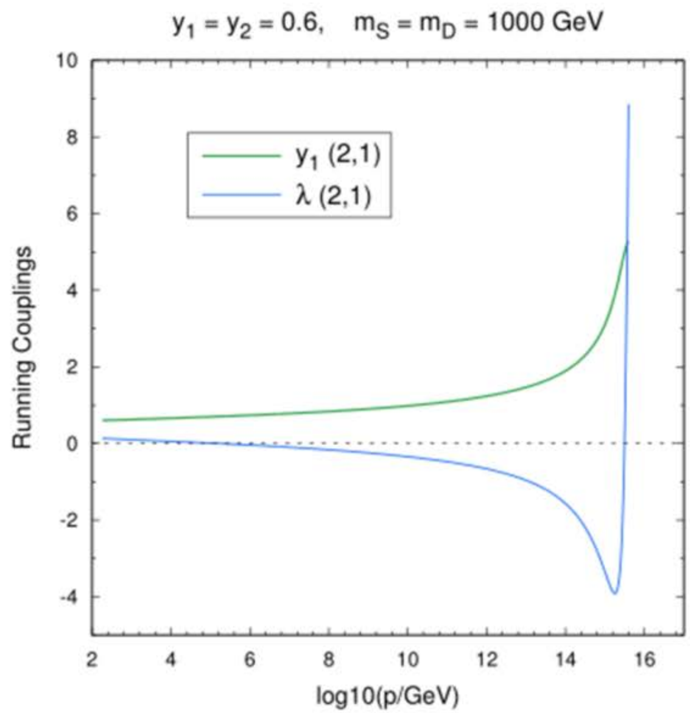


$$\beta^{SDFDM}(y_1) = \frac{y_1}{(4\pi)^2}\left[\frac{5}{2}y_1^2 + 4y_2^2 - \frac{9}{20}g_1^2 - \frac{9}{4}g_2^2 + 3y_t^2 + 3y_b^2 + y_\tau^2\right]$$

$$\beta^{SDFDM}(\lambda) = \frac{1}{(4\pi)^2}\left[-2y_1^4 - 4y_1^2y_2^2 - 2y_2^4 + 4\lambda\left(y_1^2 + y_2^2\right)\right]$$

Effect of 1-loop matching





The difference between tree-level and one-loop level matching

	$\lambda\|_{\mu=M_t}$	$\lambda_{\min}$	$\log 10(\mu/\mathrm{GeV})\|_{\lambda=0}$	$\log 10(\mu/\mathrm{GeV})\|_{\lambda=\lambda_{\min}}$	$\log 10(\mathcal{P}_0)$
SM	0.12604	-0.0147378	9.796	17.44	-539.276
BMP1(2, 1)	0.12553	-0.0093655	10.53	16.58	-987.599
BMP1(2, T)	0.12604	-0.0117875	9.991	16.49	-737.212

TABLE III. For BMP1 ($y_1 = y_2 = 0.1$, $m_S = m_D = 1000$ GeV).

Matching scale

- Since mDM is varying, we have to fix a scale to match the parameters, we choose set it at Mt
- We have checked that the effect is not very important

The effects of matching scale Q (GeV)

	$Q = M_t$	300	500	1000
$\delta^{(1)}\lambda\vert_{\text{fin}}$	$-5.1507 \cdot 10^{-4}$	$-3.5443 \cdot 10^{-4}$	$-2.0483 \cdot 10^{-4}$	$-1.8401 \cdot 10^{-6}$
$\lambda(\mu = 10^3$ GeV)	$9.6050 \cdot 10^{-2}$	$9.5885 \cdot 10^{-2}$	$9.5850 \cdot 10^{-2}$	$9.5949 \cdot 10^{-2}$
$\lambda(\mu = 10^5$ GeV)	$4.8750 \cdot 10^{-2}$	$4.8234 \cdot 10^{-2}$	$4.7890 \cdot 10^{-2}$	$4.7593 \cdot 10^{-2}$
$\lambda(\mu = 10^{10}$ GeV)	$2.2502 \cdot 10^{-3}$	$1.2939 \cdot 10^{-3}$	$5.5212 \cdot 10^{-4}$	$-2.6624 \cdot 10^{-4}$
$\lambda(\mu = 10^{16.5}$ GeV)	$-9.3648 \cdot 10^{-3}$	$-1.0404 \cdot 10^{-2}$	$-1.1238 \cdot 10^{-2}$	$-1.2201 \cdot 10^{-2}$

Vacuum stability

The present value of the vacuum-decay probability p_0 is

$$p_0 = 0.15 \frac{\Lambda_B^4}{H_0^4} e^{-S(\Lambda_B)}$$

in which:

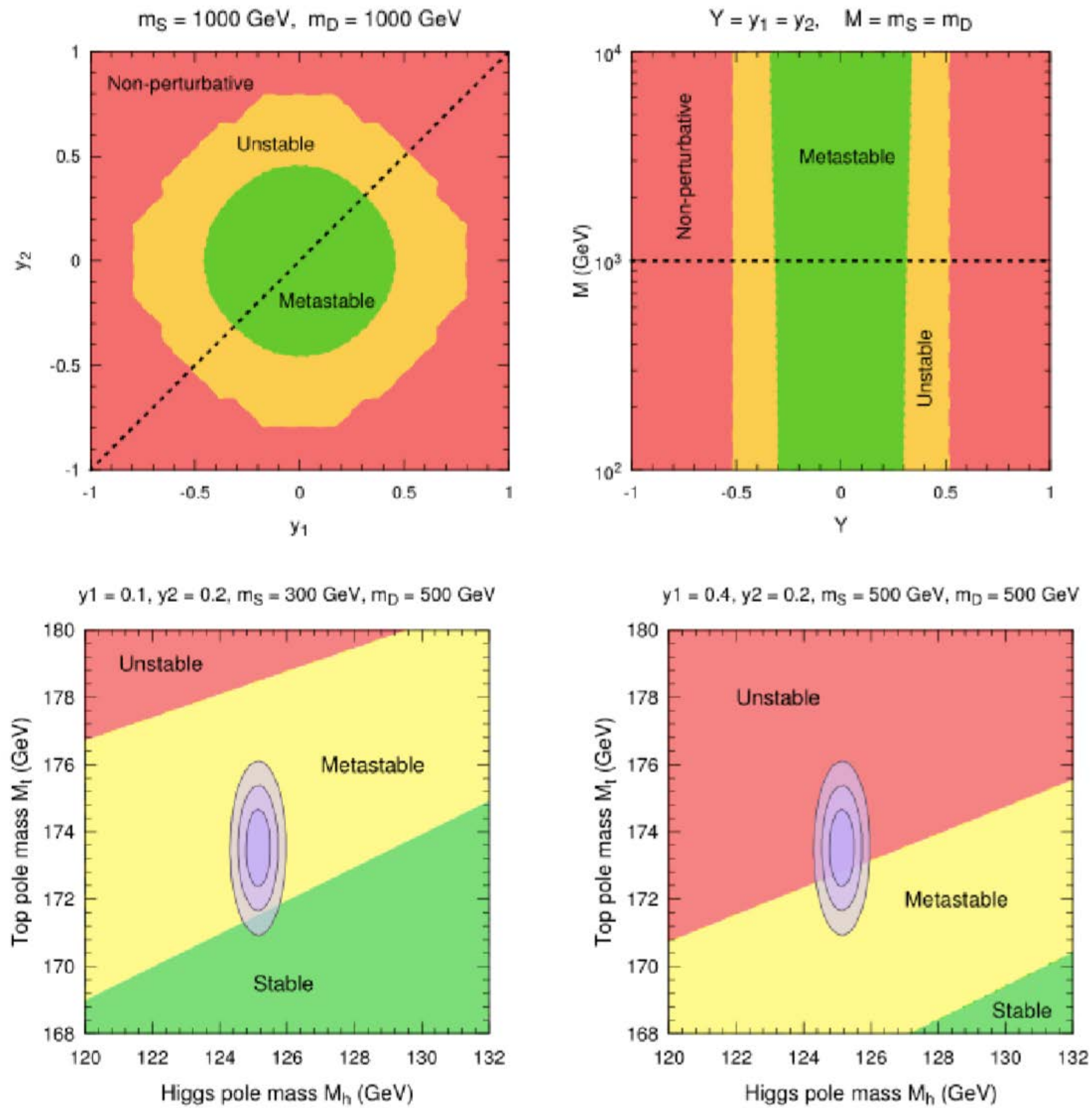
$$S(\Lambda_B) = \frac{8\pi^2}{3|\lambda(\Lambda_B)|} \qquad \beta_\lambda(\Lambda_B) = 0 \qquad H_0 \approx 67.4\,\mathrm{km/sec\ Mpc}$$

The conditions for different status of vacuum:

- Stable: if $\lambda(\Lambda_B) > 0$ until Planck scale;
- Metastable: if $\lambda(\Lambda_B) < 0$ and $\mathcal{P}_0 < 1$;
- Unstable: if $\lambda(\Lambda_B) < 0$ and $\mathcal{P}_0 > 1$;
- Non-perturbative: if $|\lambda| < 4\pi$ until Planck scale (The last condition is almost equal to demand that no Landau pole exist for the λ evolve up to Planck scale, and this can be seen

Vacuum stability in SD MD

Constrain vacuum not to be unstable set constraint on Y.



Doublet-triplet DM model

Dark sector Weyl fermions ($SU(2)_L \times U(1)_Y$):

$$D_1 = \begin{pmatrix} D_1^0 \\ D_1^- \end{pmatrix} \in \left(\mathbf{2}, -\frac{1}{2}\right), \quad D_2 = \begin{pmatrix} D_2^+ \\ D_2^0 \end{pmatrix} \in \left(\mathbf{2}, \frac{1}{2}\right), \quad T = \begin{pmatrix} T^+ \\ T^0 \\ -T^- \end{pmatrix} \in (\mathbf{3}, 0)$$

Gauge invariant kinetic terms, mass terms and Yukawa couplings:

$$\mathcal{L}_T = iT^\dagger \bar{\sigma}^\mu D_\mu T - \frac{1}{2}(m_T a_{ij} T^i T^j + \text{h.c.})$$

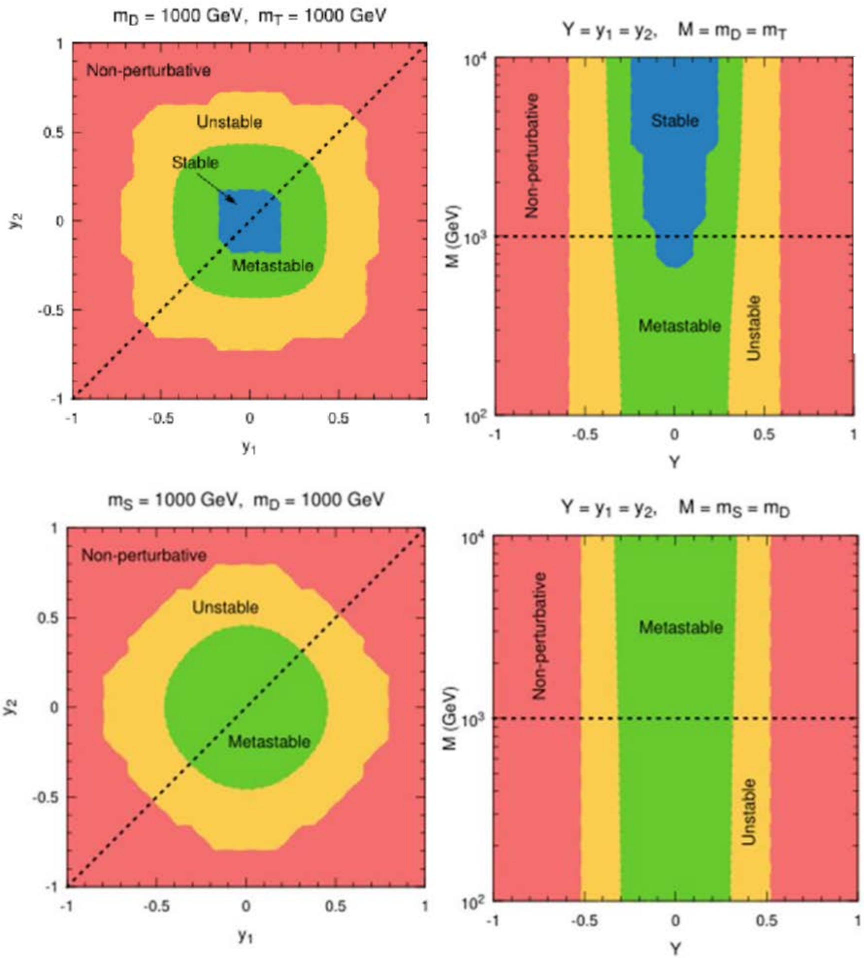
$$\mathcal{L}_D = iD_1^\dagger \bar{\sigma}^\mu D_\mu D_1 + iD_2^\dagger \bar{\sigma}^\mu D_\mu D_2 - (m_D b_{ij} D_1^i D_2^j + \text{h.c.})$$

$$\mathcal{L}_{HTD} = y_1 c_{ijk} T^i D_1^j H^k + y_2 d_{ijk} T^i D_2^j \widetilde{H}^k + \text{h.c.}$$

a_{ij}, b_{ij}, c_{ijk}, and d_{ijk} can be decoded from CG coefficients multiplied by a normalizing factor

There are four independent parameters: m_T, m_D, y_1, y_2

Vacuum stability in the DT model



$$\beta^{DTFDM}(y_1) = \frac{y_1}{(4\pi)^2}\left[\frac{11}{2}y_1^2 + 2y_2^2 - \frac{9}{20}g_1^2 - \frac{33}{4}g_2^2 + 3y_t^2 + 3y_b^2 + y_\tau^2\right]$$

$$\beta^{SDFDM}(y_1) = \frac{y_1}{(4\pi)^2}\left[\frac{5}{2}y_1^2 + 4y_2^2 - \frac{9}{20}g_1^2 - \frac{9}{4}g_2^2 + 3y_t^2 + 3y_b^2 + y_\tau^2\right]$$

Difference of beta function leads to very different results

Triplet-quadruplet DM model

Dark sector Weyl fermions ($SU(2)_L \times U(1)_Y$):

$$T = \begin{pmatrix} T^+ \\ T^0 \\ -T^- \end{pmatrix} \in (\mathbf{3}, 0), \quad Q_1 = \begin{pmatrix} Q_1^+ \\ Q_1^0 \\ Q_1^- \\ Q_1^{--} \end{pmatrix} \in \left(\mathbf{4}, -\frac{1}{2}\right), \quad Q_2 = \begin{pmatrix} Q_2^{++} \\ Q_2^+ \\ Q_2^0 \\ Q_2^- \end{pmatrix} \in \left(\mathbf{4}, \frac{1}{2}\right)$$

Gauge invariant kinetic terms, mass terms and Yukawa couplings:

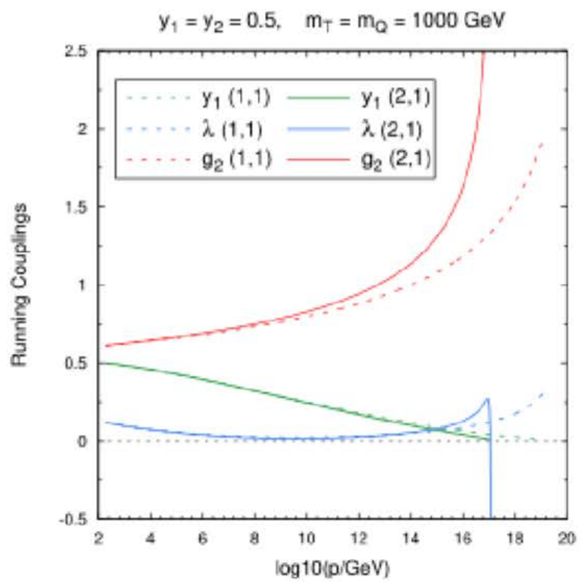
$$\mathcal{L}_T = iT^\dagger \bar{\sigma}^\mu D_\mu T - (m_T a_{ij} T^i T^j + \text{h.c.})$$

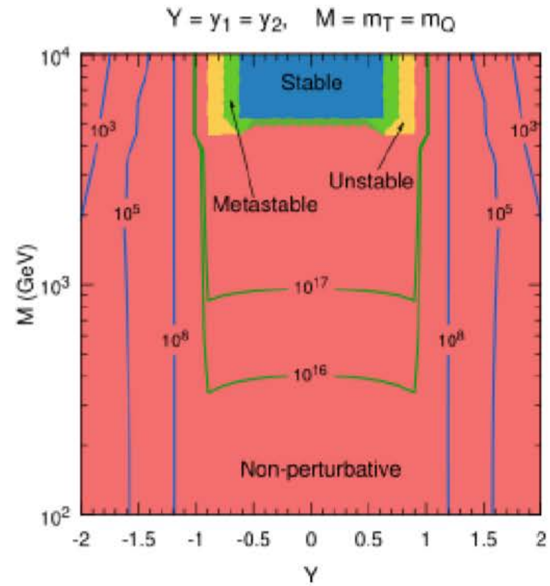
$$\mathcal{L}_Q = iQ_1^\dagger \bar{\sigma}^\mu D_\mu Q_1 + iQ_2^\dagger \bar{\sigma}^\mu D_\mu Q_2 - (m_Q b_{ij} Q_1^i Q_2^j + \text{h.c.})$$

$$\mathcal{L}_{HTQ} = y_1 c_{ijk} Q_1^i T^j H^k + y_2 d_{ijk} Q_2^i T^j \widetilde{H}^k + \text{h.c.}$$

There are four independent parameters: m_T, m_Q, y_1, y_2

Vacuum stability





$$\beta^{total}(g_2) = \beta^{SM}(g_2) + \frac{g_2^3}{(4\pi)^2}\left(\frac{\mathbf{1}}{\mathbf{2}} \times \sum_j \frac{4}{3} n_f C(r)\right) \qquad \beta^{SM}(g_2) = \frac{g_2^3}{(4\pi)^2}(-\frac{19}{6})$$

$$\beta^{SDFDM}(g_2) = \frac{g^3}{(4\pi)^2}(\frac{2}{3}), \qquad \beta^{DTFDM}(g_2) = \frac{g^3}{(4\pi)^2}(2), \qquad \beta^{TQFDM}(g_2) = \frac{g^3}{(4\pi)^2}(8)$$

High dimensional Multiplets are dangerous to make the theory nonperturbative!

Summary

- Study the EW multiplet DM models on the vacuum stability, which give strong constraints on the model parameters.
- We use 1-loop matching and 2-loop beta function; high order loop corrections are important.
- It is interesting that in some parameter space vacuum can become stable; such case deserves more study.
- High dimensional multiplets lead bad behavior at high energy and should be avoided.