Intersecting Mirror Symmetry

Yiwen Pan

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Sun Yat-sen, 2019 Jan.

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- Target: 3d $\mathcal{N}=2$ theories (on $\mathbb{R}^2\times S^1, S^3$)
- 3d $\mathcal{N}=2$ theories: known 3d **mirror symmetry**

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$$U(1)_{q,\tilde{q}}, W=0 \leftrightarrow \{X,Y,Z\}, W=XYZ\ .$$

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- 3d $\mathcal{N}=2$ theories can be generalized to **intersecting** spaces [YP, Peelaers]
- Goal (in steps):
  A. Explore 3d reduction of string S-duality
  B. Use A to predict intersecting mirror symmetry

Outline

1. Review: 3d mirror symmetry

$\mathcal{N} = 2$ theories: fields, and the Lagrangian

- Two packs of **fields**:
  - **"vector multiplet"** $(A_\mu, \sigma, \lambda, \tilde{\lambda}, D)$; gauge group G
  - **"chiral multiplet"** $(\phi, \tilde{\phi}, \psi, \tilde{\psi}, F, \tilde{F})$; representation $\mathcal{R}$
- **Lagrangians** $\mathcal{L}_{\rm VM}$ and $\mathcal{L}_{\rm CM}$

$$\mathcal{L}_{\rm VM} = \frac{ik}{4\pi} \operatorname{tr}\left[\epsilon^{\mu\nu\lambda}(A_\mu \nabla_\nu A_\lambda - \frac{2i}{3} A_\mu A_\nu A_\lambda) - (\tilde{\lambda}\lambda) + 2D\sigma\right] . \\ + \xi_{\rm FI} \operatorname{tr}\left[D - \frac{\sigma}{f}\right]$$

$$\mathcal{L}_{\rm CM} = D_\mu \tilde{\phi} D^\mu \phi - i(\tilde{\psi}\gamma^\mu D_\mu \psi) + \frac{\mathcal{R}}{8}\tilde{\phi}\phi + i(\tilde{\psi}\sigma\psi) \\ + i(\tilde{\psi}\lambda)\phi - i\tilde{\phi}(\tilde{\lambda}\psi) + \tilde{\phi}D\phi + \tilde{\phi}\sigma^2\phi + \tilde{F}F + W .$$

Parameters

- Gauge group and matter representation $\mathcal{R}$
- Chern-Simons level k ($k = 0$ for us)

- **Fayet–Iliopoulos parameter** ξ_{FI}
- **complex masses** m_i: from shifting $\sigma\phi_i \to (\sigma + m_i)\phi_i$
- **Superpotential W** constrains the masses and $U(1)_{\mathcal{R}}$ charges of chirals

Mirror symmetry

- IR duality: mirror dual theories flow to same fixed point
- For $\mathcal{N} = 4$:
  - $\mathcal{R}$-symmetry groups $SU(2)_{\mathrm{H}} \overset{\text{mirror}}{\longleftrightarrow} SU(2)_{\mathrm{C}}$
  - moduli spaces $\mathcal{M}_{\mathrm{H}} \overset{\text{mirror}}{\longleftrightarrow} \mathcal{M}_{\mathrm{C}}$
  - chiral rings $\chi_{\mathrm{H}} \overset{\text{mirror}}{\longleftrightarrow} \chi_{\mathrm{C}}$
  - FI-params $\overset{\text{mirror}}{\longleftrightarrow}$ masses
- Mirror dual theories: **equal partition functions**
  (Important tool)

2. Review: Type IIB S-duality

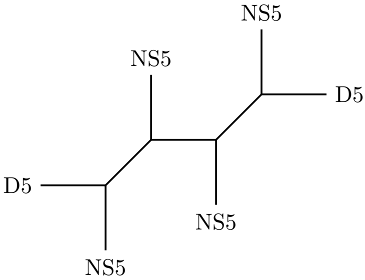
IIB string theory

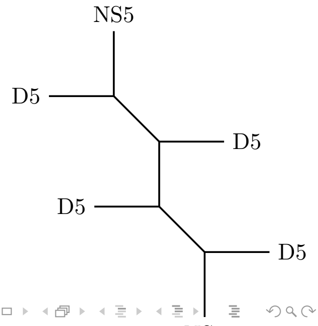
- Simple branes: **D5** and **NS5** branes
- Bound state: **(p,q)-brane** $\sim p$ D5 $+ q$ NS5

  $(1,0)$-brane $=$ D5 brane

  $(0,1)$-brane $=$ NS5 brane

- Lots of (p,q) branes (and some rules): **(p,q)-web**





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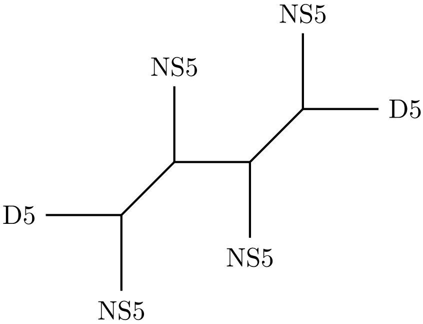
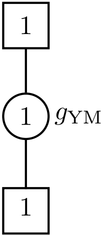
  - special webs = special 5d $\mathcal{N} = 1$ theories





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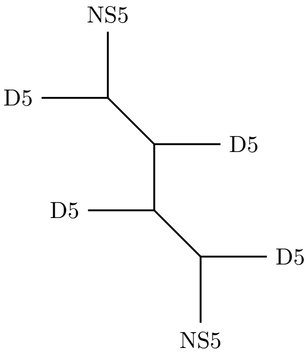

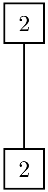

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  [Aganagic, et.al.][Iqbal, Kozçaz, Vafa][Awata, Kanno]

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- Topological vertex

$$\text{Momentum } p \to \text{Young diagrams } Y$$

$$\int dp \to \sum_{Y}$$

$$\text{interaction vertex} \to \text{product of Macdonald function } P_{\lambda/\mu}$$

$$\text{internal lines} \to \text{"framing factors", "Kahler params" } Q$$

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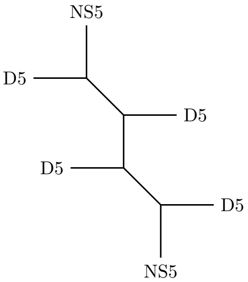
$Z\Big[$ 

$\Big] = \dfrac{(Q\ldots;q,t^{-1})(Q\ldots;q,t^{-1})}{\prod(Q\ldots p^{1/2};q,t^{-1})}$

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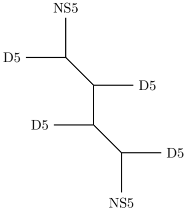

where the **double q-Pochhammer symbol**

$$(x;q,t^{-1}) \equiv \prod_{m,n=0}^{+\infty} (1 - xq^m t^{-n}) \ .$$

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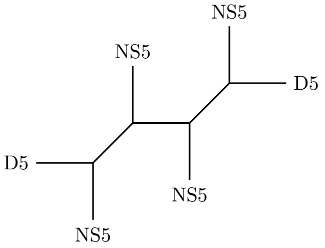

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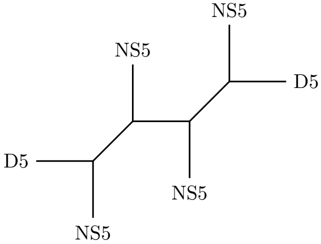
$Z\Big[$ 

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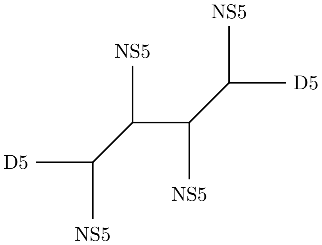

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- $\sum_Y$: sum over all Young diagrams
- $Z_{\text{inst}}(Y;Q)$: some rational function of Q's.

Webs and "Feynman diagrams"

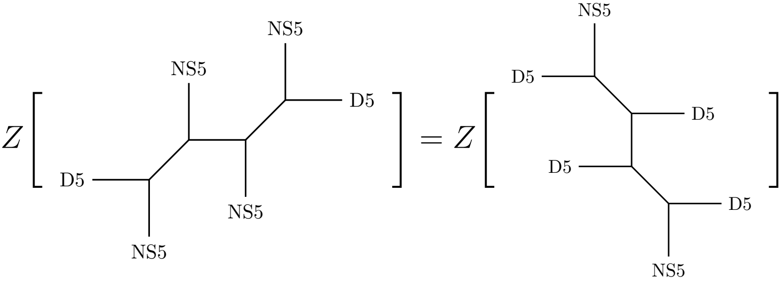
- Q's: 5d **gauge couplings** g_{YM} and **masses** M's; Schematically,

$$Q\ {}'\mathrm{s} \sim e^{2\pi i M} e^{\frac{1}{g_{\mathrm{YM}}^2}} \ .$$

S-duality

S-duality of string theory:

- NS5 $\leftrightarrow$ D5
- $(p, q) \leftrightarrow (q, -p)$
- non-trivial equality betweens Z's



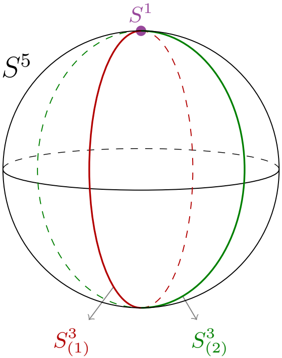
3. S-duality to 3d Mirror duality

Higgsing and defects

- 5d theory $\xrightarrow{\text{Higgsing}}$ 5d/3d/1d coupled system

  [YP, Peelaers, 2017][Nieri, YP, Zabzine, 2018]

- 3d/1d sector (5d p.o.v): **supersymmetric defects**



$$S^5: \quad \omega_1^2|z_1|^2+\omega_2^2|z_2|^2+\omega_3^2|z_3|^2=1$$

$$S^3_{(2)}: \quad \omega_1^3|z_1|^2+ \qquad +\omega_3^2|z_3|^2=1$$

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  - Additional 1d theory $\mathcal{T}^{\text{1d}}$ on S^1
  - $\mathcal{T}^{(1)}$ and $\mathcal{T}^{(2)}$ interact with $\mathcal{T}^{\text{1d}}$ (capturing intersection).

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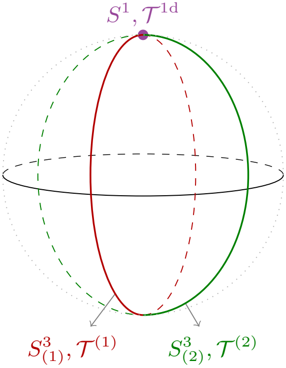

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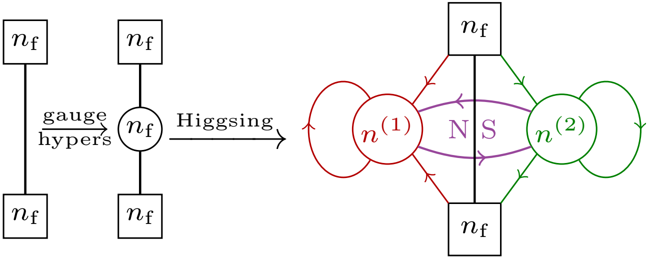

$\mathcal{T}$ $\widetilde{\mathcal{T}}$ $\mathcal{T}^{S^3_{(1)} \cup S^3_{(2)} \subset S^5}$

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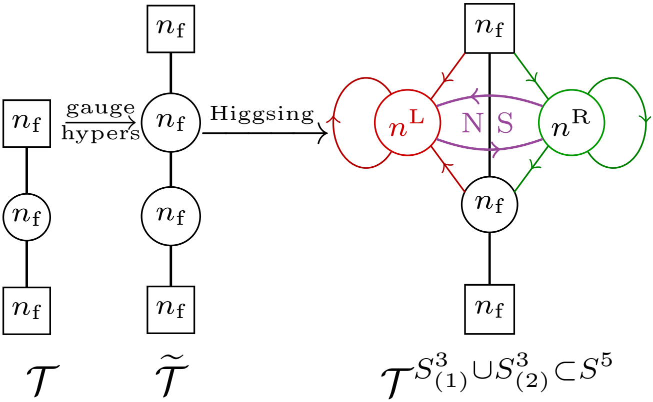

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- The residue = **p. f. with defects**

$$\mathrm{Res}_{M \to M^{\vec{n}}} Z(M) = Z^{S^3_{(1)} \cup S^3_{(2)}}$$

(a) **Factorizing instanton partition function**

summands [YP, Peelaers]: $Z_{\mathrm{inst}} \sim Z_{\mathrm{vort}} Z_{\mathrm{vort}} Z_{\mathrm{intersection}}$

(b) $|\sum_Y|_{SL(3,\mathbb{Z})} \to$ Jeffrey-Kirwan residue

Higgsing the partition function

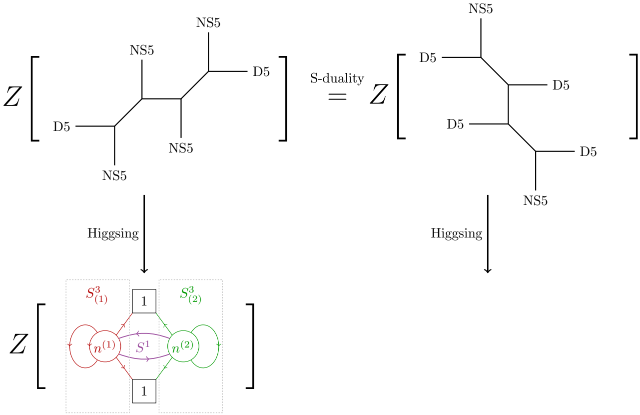
The **p. f.** of **intersecting 3d gauge theory**

$$Z^{S^3_{(1)} \cup S^3_{(2)}} = \int d^{n^{(1)}}\sigma d^{n^{(2)}}\sigma Z^{S^3_{(1)}}(\sigma^{(1)}) Z^{S^3_{(2)}}(\sigma^{(2)}) Z^{S^1}(\sigma^{(1)}, \sigma^{(2)}) \ .$$

- 5d gauge coupling $g_{\text{YM}}^{-2} \xrightarrow{\text{Higgsing}}$ FI-parameter ξ_{FI}
- 5d masses $\xrightarrow{\text{Higgsing}}$ 3d massses
- 5d/3d superpotentials generated: **mass relations** across theories on $S^3_{(1)}$, $S^3_{(2)}$

Reducing S-duality

- Start from S-dual 5d partition functions
- Apply Higgsing on both sides
- S-duality $\xrightarrow{\text{Higgsing}}$ induced 3d **mirror?** duality



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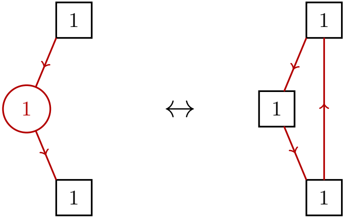

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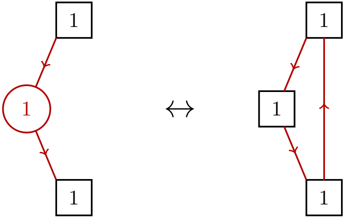

$$U(1)_{q,\tilde{q}}, W = 0 \;\leftrightarrow \{x, y, z\}, W = xyz \;.$$

- **Recovered well-known 3d mirror pair on $S^3_{(1)}$**

Reducing S-duality ($n^{(2)} = 0$)

- $n^{(1)} = n > 1$ [Nieri, YP, Zabzine]
  - LHS: $U(n)_{q,\tilde{q},\Phi}, W = 0$ on $S^3_{(1)}$
  - RHS: Collection of free chirals on $S^3_{(1)}$

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- $U(n)$ theory (LHS) has **topological** $U(1)$ global symmetry:
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  - **gauge it** (on both sides) (integrate over ξ_{FI})
  - LHS: $U(n) \to SU(n)$
  - RHS: free chirals $\to U(1)$ gauge theory

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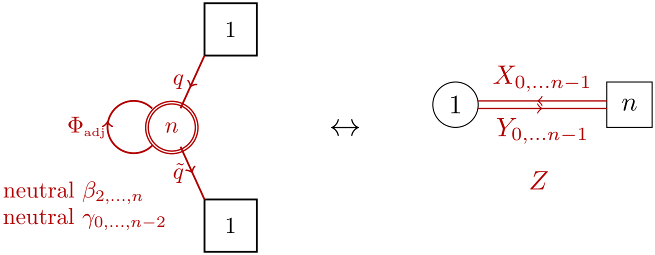
- After gauging (and rearrangement of neutral chirals)



$W = \sum_{\mu=2}^{n} \beta_\mu \Phi^\mu + \sum_{\mu=0}^{n-2} \gamma_\mu \tilde{q} \Phi^\mu q$

$W = \sum_{\mu=0}^{n-1} X_\mu Y_\mu Z$

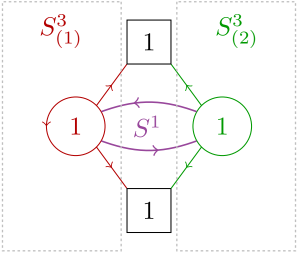
Reducing S-duality ($n^{(2)} = 0$)

- $n^{(1)} = n > 1$ [Nieri, YP, Zabzine]
  - LHS: $U(n)_{q,\tilde{q},\Phi}, W = 0$ on $S^3_{(1)}$
  - RHS: Collection of free chirals on $S^3_{(1)}$
- After gauging
  - known **mirror pair**
  - Important in studying **UV Lagrangian description** and SUSY enhancement of **Argyres-Douglas theories**

    [Benvenuti, Giacomelli]

Reducing S-duality ($n^{(2)} > 0$)

- Conjecture [Nieri, YP, Zabzine]: when $n^{(1)}, n^{(2)} \geq 1$, S-duality reduces to mirror symmetry between a set of intersecting gauge theories on $S^3_{(1)}$ and $S^3_{(2)}$.
  e.g.



$$\leftrightarrow \quad (XYZ)_{(1)} \cdots\cdots (XYZ)_{(2)}$$

Reducing S-duality ($n^{(2)} > 0$)

- Conjecture [Nieri, YP, Zabzine]: when $n^{(1)}, n^{(2)} \geq 1$, S-duality reduces to mirror symmetry between a set of intersecting gauge theories on $S^3_{(1)}$ and $S^3_{(2)}$.

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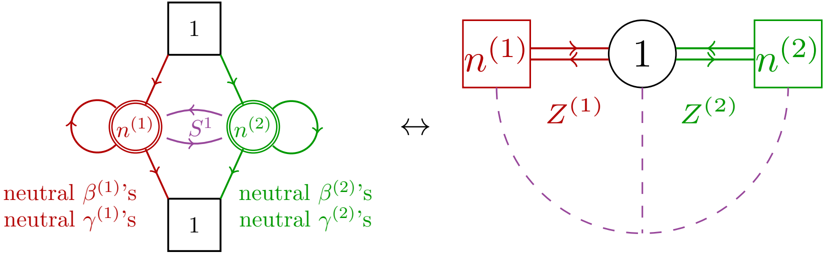

(superpotentials omitted from the figure)

Reducing S-duality ($n^{(2)} > 0$)

Or as integral identity,

$$Z^{\mathbb{S}^3_{(1)} \cup \mathbb{S}^3_{(2)}}_{\mathrm{U}(n^{(1)})\text{-SQCDA} \,\cup\, \mathrm{U}(n^{(2)})\text{-SQCDA}} =$$

$$= \prod_{\alpha=1}^{2} \left[\mathrm{e}^{-\pi i n \zeta (m+\tilde{m})} \prod_{\mu=0}^{n-1} s_b\left(\frac{iQ}{2} + m_{\Phi\mu}\right) s_b\left(\frac{iQ}{2} + m_{Z\mu}\right) s_b\left(\frac{iQ}{2} + m_{X\mu}\right) s_b\left(\frac{iQ}{2} + m_{Y\mu}\right) \right]_{(\alpha)} \times$$

$$\times \prod_{\mu=0}^{n^{(1)}-1} \prod_{\nu=0}^{n^{(2)}-1} \frac{\sin \frac{\pi i}{2}\left(b_{(1)}(m_{X\mu} - m_{\Phi\mu})^{(1)} + b_{(2)}(m_{X\mu} - m_{\Phi\nu})^{(2)} + ib^2_{(2)} + ib^2_{(1)}\right)\left(X \to Y\right)}{\sin \pi i \left(b_{(1)} m^{(1)}_{\Phi\mu} + b_{(2)} m^{(2)}_{\Phi\nu}\right) \sin \frac{\pi i}{2}\left(b_{(1)}(m_{Z\mu} + m_{\Phi\mu})^{(1)} + b_{(2)}(m_{Z\nu} + m_{\Phi\nu})^{(2)}\right)} ,$$

and its corollaries.

4. Summary and Outlook

Summary

- 5d theories have **S-duality**
- 5d $\xrightarrow{\text{Higgsing}}$ 3d intersecting gauge theories
- S-duality $\xrightarrow{\text{Higgsing}}$ known 3d mirror symmetry
- S-duality $\xrightarrow{\text{Higgsing}}$ general 3d mirror symmetry on intersecting spaces (1d chiral $\xrightarrow{\text{mirror}}$ 1d Fermi)

Outlook

- Generalize other known dualities (Seiberg, 3d-3d) to intersecting spaces
- Explore algebraic origin of 3d mirror symmetry
  - 3d gauge theories $\leftrightarrow$ q-Virasoro algebra (VEV of integrated product of screening charges)
    [Nedelin, Nieri, Zabzine][Nieri, YP, Zabzine] ...
  - 5d gauge theories $\leftrightarrow$ DIM algebra, q-Virasoro algebra
    [Awata, Pestun][Awata, et.al.][Bourgine] ...
  - S-duality $\leftrightarrow$ DIM $SL(2,\mathbb{Z})$ automorphism
    [Bourgine] ...
  - **mirror symmetry $\leftrightarrow$ DIM algebra automorphism**?

Thank you!