Wigner Function and Quantum Transport Equation

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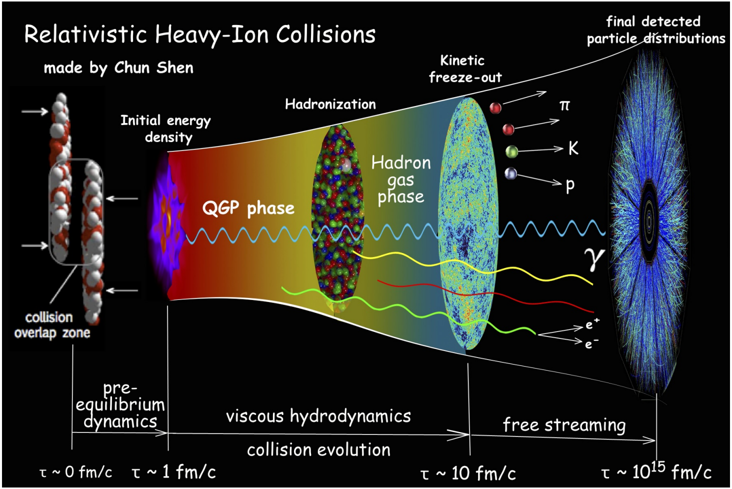
Particle 2019 Meeting

Outline

Relativistic Heavy-Ion Collisions



Anomaly Transport

For chiral fermions

$$\vec{J} \sim q\mu_A \vec{B} + n\mu_A\mu_B \vec{\omega}$$

nature physics

Letter | Published: 08 February 2016

Chiral magnetic effect in $ZrTe_5$

Qiang Li, Dmitri E. Kharzeev, Cheng Zhang, Yuan Huang, I. Pletikosić, A. V. Fedorov, R. D. Zhong, J. A. Schneeloch, G. D. Gu & T. Valla

Nature Physics **12**, 550–554 (2016) | Download Citation

Anomaly Transport

Ways to study anomaly transport:

- Fluid Dynamics.
- Transport Theory.

Why transport theory?

- Non-equilibrium.
- Inhomogeneous.

What theory?

- Classical: distribution function – Boltzman equation.
- Quantum: Wigner function – quantum transport theory.

Covariant and Equal-time Wigner Function

Covariant Wigner operator:

$$\hat{W}(x,p) = \int \mathrm{d}^4 y e^{ip\cdot y}\Psi(x+\frac{y}{2})e^{iQ\int_{1/2}^{1/2}\mathrm{d}sA(x+sy)\cdot y}\bar{\Psi}(x-\frac{y}{2}).$$

Equal-time Wigner operator:

$$\begin{aligned}\hat{W}(x,\vec{p}) &= \int \mathrm{d}^3 y e^{i\vec{p}\cdot\vec{y}}\Psi(t,\vec{x}+\frac{\vec{y}}{2})e^{iQ\int_{1/2}^{1/2}\mathrm{d}s\vec{A}(x+sy)\cdot\vec{y}}\Psi^\dagger(t,\vec{x}-\frac{\vec{y}}{2}) \\ &= \int \mathrm{d}p_0 W(x,\vec{p})\gamma_0.\end{aligned}$$

Wigner function:

$$W = \langle \hat{W} \rangle.$$

Massless Fermion in Electromagnetic Field

Lagrangian:

$$\mathcal{L}_\chi = \Psi^\dagger(i\mathrm{D}_0 + i\chi\vec{\sigma}\cdot\vec{\mathrm{D}})\Psi.$$

Field equations:

$$(iD_0 + i\chi\vec{\sigma}\cdot\vec{\mathrm{D}})\Psi = 0.$$

Wigner Operator

$$\hat{W}(x,\vec{p}) = \int \mathrm{d}^3 y e^{i\vec{p}\cdot\vec{y}}\Psi(t,\vec{x}+\frac{\vec{y}}{2})e^{iQ\int_{1/2}^{1/2}\mathrm{d}s\vec{A}(x+sy)\cdot\vec{y}}\Psi^\dagger(t,\vec{x}-\frac{\vec{y}}{2}).$$

Kinetic equation

Kinetic equation for covariant Wigner operator (and function):

$$
\begin{aligned}
(K_0 - \chi\sigma_i K_i)W &= 0 \\
K_\mu &= \Pi_\mu + \frac{i\hbar}{2}\mathrm{D}_\mu \\
\Pi_\mu &= p_\mu - iQ\hbar\int_{-\frac{1}{2}}^{\frac{1}{2}} \mathrm{d}s s F_{\mu\nu}(x - i\hbar s\partial_p)\partial_p^\nu \\
\mathrm{D}_\mu &= \partial_\mu - Q\int_{-\frac{1}{2}}^{\frac{1}{2}} \mathrm{d}s F_{\mu\nu}(x - i\hbar s\partial_p)\partial_p^\nu
\end{aligned}
$$

Spin Decomposition

W is Hermite, so

$$
\begin{aligned}
W(x,p) &= \frac{1}{2}(F + \chi \sum_i \sigma_i V_i) \\
W(x,\vec{p}) &= \frac{1}{2}(f + \chi \vec{\sigma} \cdot \vec{g}).
\end{aligned}
$$

Kinetic equations for f and $\vec{g}$:

$$
\begin{aligned}
\hbar(\mathrm{D}_t f + \vec{\mathrm{D}} \cdot \vec{g}) &= 0 \\
\hbar(\mathrm{D}_t \vec{g} + \vec{\mathrm{D}} f) - 2\chi \vec{\Pi} \times \vec{g} &= 0 \\
\int \mathrm{d}p_0 p_0 F - \bar{\Pi}_0 f - \vec{\Pi} \cdot \vec{g} &= 0 \\
\int \mathrm{d}p_0 p_0 \vec{V} - \bar{\Pi}_0 \vec{g} - \vec{\Pi} f - \chi \frac{\hbar}{2} \vec{\mathrm{D}} \times \vec{g} &= 0
\end{aligned}
$$

Semi-classical expansion

Use semi-classical expansion to try to solve the equations. Up to the first oder of $\hbar$, W is on-shell.
So we assume:

$$\begin{aligned} f^{(0)} &= (f^{(0)+} + f^{(0)-}) \\ f^{(1)} &= (f^{(1)+} + f^{(1)-}) \\ \int \mathrm{d}p_0 p_0 F^{(0)} &= E_p(f^{(0)+} - f^{(0)-}) \\ \int \mathrm{d}p_0 p_0 F^{(1)} &= E_p(f^{(1)+} - f^{(1)-}) + E^{(1)+} f^{(0)+} + E^{(1)-} f^{(0)-} \end{aligned}$$

Similarly:

$$\int \mathrm{d}p_0 p_0 \vec{V}^{(1)} = E_p(\vec{g}^{(1)+} - \vec{g}^{(1)-}) + \vec{E}^{(1)+} f^{(0)+} + \vec{E}^{(1)-} f^{(0)-}$$

$\hbar$ Expansion

$$
\begin{aligned}
\int \mathrm{d}p_0 F^{(1)} &= E_p(f^{(1)+} - f^{(1)-}) + E^{(1)+} f^{(0)+} + E^{(1)-} f^{(0)-} \\
\int \mathrm{d}p_0 \vec{V}^{(1)} &= E_p(\vec{g}^{(1)+} - \vec{g}^{(1)-}) + \vec{E}^{(1)+} f^{(0)+} + \vec{E}^{(1)-} f^{(0)-}
\end{aligned}
$$

This is in consistent with the assumption that:

$$
W(x,p) = \delta(p^2 - E^2 - \hbar\Delta E)\tilde{W}
$$

0th Order Equations

$$\begin{aligned}
\vec{p} \times \vec{g}^{(0)\pm} &= 0 \\
E_p^{(0)} f^{(0)\pm} \mp \vec{p} \cdot \vec{g}^{(0)\pm} &= 0 \\
E_p^{(0)} \vec{g}^{(0)\pm} \mp \vec{p} f^{(0)\pm} &= 0 \\
(\partial_t + Q\vec{E} \cdot \vec{\partial}_p) f^{(0)\pm} - (\vec{\partial} + Q\vec{B} \times \vec{\partial}_p) \cdot \vec{g}^{(0)\pm} &= 0
\end{aligned}$$

Solution:

$$\begin{aligned}
\vec{g}^{(0)\pm} &= \pm \frac{\vec{p}}{E_p^{(0)}} f^{(0)\pm} \\
E_p^{(0)} &= p
\end{aligned}$$

1st Order Equations

$$
\begin{aligned}
(\partial_t + Q\vec{E}\cdot\vec{\partial}_p)\vec{g}^{(0)\pm} - (\vec{\partial} + Q\vec{B}\times\vec{\partial}_p)f^{(0)\pm} + 2\chi\vec{p}\times\vec{g}^{(1)\pm} &= 0\\
E_p^{(0)}f^{(1)\pm} + E_p^{(1)\pm}f^{(0)\pm} \mp \vec{p}\cdot\vec{g}^{(1)\pm} &= 0\\
E_p^{(0)}\vec{g}^{(1)\pm} + \vec{E}_p^{(1)\pm}f^{(0)\pm} \mp \vec{p}f^{(1)\pm} \mp \chi\frac{1}{2}(\vec{\partial} + Q\vec{B}\times\vec{\partial}_p)\times\vec{g}^{(0)\pm} &= 0\\
(\partial_t + Q\vec{E}\cdot\vec{\partial}_p)f^{(1)\pm} + (\vec{\partial} + Q\vec{B}\times\vec{\partial}_p)\cdot\vec{g}^{(1)\pm} &= 0
\end{aligned}
$$

Solution to 1st Order Equations

$$
\begin{aligned}
E_p^{(1)\pm} &= \mp\chi\frac{Q\vec{B}\cdot\vec{p}}{2p^2} \\
\vec{E}_p^{(1)\pm} &= \mp\chi\frac{Q\vec{E}\times\vec{p}}{2p^2} - \chi\frac{Q\vec{B}}{2p} \\
f^{(1)\pm} &= f_\chi^{(1)\pm} \pm \chi\frac{Q\vec{B}\cdot\vec{p}}{2p^3} f^{(0)\pm} \\
\vec{g}^{(1)\pm} &= \pm\frac{\vec{p}}{p} f_\chi^{(1)\pm} - \frac{\chi\vec{p}}{2p^2}\times(\vec{\partial} + Q\vec{B}\times\vec{\partial}_p) f^{(0)\pm} \pm \chi\frac{Q\vec{E}\times\vec{p}}{2p^3} f^{(0)\pm}
\end{aligned}
$$

There is arbitrariness in the solution, which corresponds to the selection of reference frame.

Transport Equations

Up to the first order of *hbar*, the transport equation for$f_\chi^{\pm} = f^{(0)\pm} + f_\chi^{(1)\pm}$ is

$$
\begin{aligned}
&(1 \pm \hbar\chi \frac{Q\vec{B}\cdot\vec{p}}{2p^3})(\partial_t + Q\vec{E}\cdot\vec{\partial}_p) f_\chi^{\pm} + \chi\hbar\frac{Q}{2p^2}[\vec{\partial}(\vec{B}\cdot\vec{p})\cdot\vec{\partial}_p] f^{\pm} \\
&\pm[\frac{\vec{p}}{p}(1 \pm \hbar\chi\frac{Q\vec{B}\cdot\vec{p}}{p^3}) + \hbar\chi\frac{Q\vec{E}\times\vec{p}}{2p^3}]\cdot(\vec{\partial} + Q\vec{B}\times\vec{\partial}_p) f_\chi^{\pm} = 0
\end{aligned}
$$

Berry Curvature

Define $\vec{b} = \pm\chi\frac{\vec{p}}{2p^3}$, $E_p = p(1 - \hbar Q\vec{B}\cdot\vec{b})$,
$\vec{v} = \pm\frac{\vec{p}}{p}(1 + 2\hbar Q\vec{B}\cdot\vec{b}) - \hbar\frac{Q\vec{B}}{2p^2} = \frac{\partial E_p}{\vec{\partial} p}$,
the equation can be expressed as:

$$
\begin{aligned}
(1 + \hbar Q\vec{B}\cdot\vec{b})\partial_t f + [\vec{v} + \hbar(\vec{v}\cdot\vec{b})Q\vec{B} + \hbar Q\vec{E}\times\vec{b}]\cdot\vec{\partial} f & \\
+[\vec{v}\times Q\vec{B} + Q\vec{E} + \hbar Q\vec{E}\cdot Q\vec{B}\vec{b} - \hbar\vec{\partial} E_p]\cdot\vec{\partial}_p f &= 0
\end{aligned}
$$

This is in consistent with former results (Son and Yamamoto, Phys. Rev. D 87, 085016).

Transport Theory for Massive Fermions

HIC in reality:

- Fluctuation
- Non-equilibrium to equilibrium
- Finite mass quarks

We need a transport theory for massive fermions.

NJL Lagrangian

The NJL Lagrangian density:

$$\mathcal{L} = \bar{\psi}\,(i\gamma^{\mu}D_{\mu} - m_0)\,\psi + G\sum_{a=0}^{3}\left[(\bar{\psi}\tau_a\psi)^2 + (\bar{\psi}i\gamma_5\tau_a\psi)^2\right]$$

Condensates:

$$\begin{aligned}\sigma(x) &= 2G\langle\bar{\psi}\psi\rangle \\ \pi(x) &= 2G\langle\bar{\psi}i\gamma_5\tau_3\psi\rangle\end{aligned}$$

Taking mean field approximation, the Lagrangian becomes:

$$\mathcal{L} = \bar{\psi}\,[i\gamma^{\mu}D_{\mu} + i\gamma_5\tau_3\pi(x) - (m_0 - \sigma(x))]\,\psi$$

Dirac equation:

$$[i\gamma^{\mu}D_{\mu} + i\gamma_5\tau_3\pi(x) - (m_0 - \sigma(x))]\,\psi = 0$$

Transport Equations for All Components

Using the Dirac equation, we have:

$$
\begin{aligned}
\hbar(\mathrm{D}_t f_0 + \vec{\mathrm{D}}\vec{g}_1) &= 2\pi_o f_2 - 2\sigma_o f_3 \\
\hbar(\mathrm{D}_t f_1 + \vec{\mathrm{D}}\cdot\vec{g}_0) &= 2\pi_e f_3 - 2(m-\sigma_e) f_2 \\
\hbar\mathrm{D}_t f_2 - 2\vec{\Pi}\cdot\vec{g}_3 &= 2\pi_o f_0 + 2(m-\sigma_e) f_1 \\
\hbar\mathrm{D}_t f_3 - 2\vec{\Pi}\cdot\vec{g}_2 &= -2\pi_e f_1 - 2\sigma_o f_0 \\
\hbar(\mathrm{D}_t \vec{g}_0 + \vec{\mathrm{D}} f_1) - 2\vec{\Pi}\times\vec{g}_1 &= 2\pi_o\vec{g}_2 - 2\sigma_o\vec{g}_3 \\
\hbar(\mathrm{D}_t \vec{g}_1 + \vec{\mathrm{D}} f_0) - 2\vec{\Pi}\times\vec{g}_0 &= -2\pi_e\vec{g}_3 - 2(m-\sigma_e)\vec{g}_2 \\
\hbar(\mathrm{D}_t \vec{g}_2 - \vec{\mathrm{D}}\times\vec{g}_3) + 2\vec{\Pi} f_3 &= 2\pi_0\vec{g}_0 + 2(m-\sigma_e)\vec{g}_1 \\
\hbar(\mathrm{D}_t \vec{g}_3 + \vec{\mathrm{D}}\times\vec{g}_2) + 2\vec{\Pi} f_2 &= 2\pi_e\vec{g}_1 + 2\sigma_o\vec{g}_0
\end{aligned}
$$

Transport Equations for All Components

$$\int \mathrm{d}p_0 p_0 F + \tilde{\Pi}_0 f_3 - \frac{\hbar}{2}\vec{\mathrm{D}}\cdot\vec{g}_2 = \pi_o f_1 + (m-\sigma_e) f_0$$

$$\int \mathrm{d}p_0 p_0 P + \tilde{\Pi}_0 f_2 - \frac{\hbar}{2}\vec{\mathrm{D}}\cdot\vec{g}_3 = \pi_e f_0 + \sigma_o f_1$$

$$\int \mathrm{d}p_0 p_0 V_0 + \tilde{\Pi}_0 f_0 - \vec{\Pi}\cdot\vec{g}_1 = \pi_e f_2 + (m-\sigma_e) f_3$$

$$\int \mathrm{d}p_0 p_0 V_i + \tilde{\Pi}_0 \vec{g}_1 - \vec{\Pi} f_0 - \frac{\hbar}{2}\vec{\mathrm{D}}\times\vec{g}_0 = \pi_o \vec{g}_3 + \sigma_o \vec{g}_2$$

$$\int \mathrm{d}p_0 p_0 A_0 - \tilde{\Pi}_0 f_1 + \vec{\Pi}\cdot\vec{g}_0 = -\pi_o f_3 + \sigma_o f_2$$

$$\int \mathrm{d}p_0 p_0 A_i - \tilde{\Pi}_0 \vec{g}_0 + \vec{\Pi} f_1 + \frac{\hbar}{2}\vec{\mathrm{D}}\times\vec{g}_1 = -\pi_e \vec{g}_2 + (m-\sigma_e)\vec{g}_3$$

$$\int \mathrm{d}p_0 p_0 S_{0i} - \tilde{\Pi}_0 \vec{g}_2 - \vec{\Pi}\times\vec{g}_3 - \frac{\hbar}{2}\vec{\mathrm{D}} f_3 = -\pi_e \vec{g}_0 - \sigma_o \vec{g}_1$$

$$\int \mathrm{d}p_0 p_0 \epsilon_{ijk} S^{jk} - 2\tilde{\Pi}_0 \vec{g}_3 + 2\vec{\Pi}\times\vec{g}_2 - \vec{\mathrm{D}} f_2 = 2\pi_0 \vec{g}_1 + 2(m-\sigma_e)\vec{g}_0$$

0th Order Equations

0th order equations:

$$\begin{aligned}
0 &= \pi f_{3a}^{(0)\pm} - (m-\sigma) f_{2a}^{(0)\pm}, \\
-\vec{p}\cdot\vec{g}_{3a}^{(0)\pm} &= (m-\sigma) f_{1a}^{(0)\pm}, \\
-\vec{p}\cdot\vec{g}_{2a}^{(0)\pm} &= -\pi f_{1a}^{(0)\pm}, \\
-\vec{p}\times\vec{g}_{1a}^{(0)\pm} &= 0, \\
-\vec{p}\times\vec{g}_{0a}^{(0)\pm} &= -\pi\vec{g}_{3a}^{(0)\pm} - (m-\sigma)\vec{g}_{2a}^{(0)\pm}, \\
\vec{p} f_{3a}^{(0)\pm} &= (m-\sigma)\vec{g}_{1a}^{(0)\pm}, \\
\vec{p} f_{2a}^{(0)\pm} &= \pi\vec{g}_{1a}^{(0)\pm}.
\end{aligned}$$

condensates:

$$\sigma^{(0)\pm}(x) = G\int \frac{d^3\vec{p}}{(2\pi)^3}\left(f_{3u}^{(0)\pm}(x,\vec{p}) + f_{3d}^{(0)\pm}(x,\vec{p})\right),$$

$$\pi^{(0)\pm}(x) = -G\int \frac{d^3\vec{p}}{(2\pi)^3}\left(f_{2u}^{(0)\pm}(x,\vec{p}) - f_{2d}^{(0)\pm}(x,\vec{p})\right).$$

0th Order Equations

$$
\begin{aligned}
\pm E_p f_{3a}^{(0)\pm} &= (m-\sigma) f_{0a}^{(0)\pm}, \\
\pm E_p f_{2a}^{(0)\pm} &= \pi f_{0a}^{(0)\pm}, \\
\pm E_p f_{0a}^{(0)\pm} - \vec{p}\cdot\vec{g}_{1a}^{(0)\pm} &= \pi f_{2a}^{(0)\pm} + (m-\sigma) f_{3a}^{(0)\pm}, \\
\pm E_p \vec{g}_{1a}^{(0)\pm} - \vec{p} f_{0a}^{(0)\pm} &= 0, \\
\mp E_p f_{1a}^{(0)\pm} + \vec{p}\cdot\vec{g}_{0a}^{(0)\pm} &= 0, \\
\mp E_p \vec{g}_{0a}^{(0)\pm} + \vec{p} f_{1a}^{(0)\pm} &= -\pi \vec{g}_{2a}^{(0)\pm} + (m-\sigma)\vec{g}_{3a}^{(0)\pm}, \\
\mp E_p \vec{g}_{2a}^{(0)\pm} - \vec{p}\times\vec{g}_{3a}^{(0)\pm} &= -\pi \vec{g}_{0a}^{(0)\pm}, \\
\mp E_p \vec{g}_{3a}^{(0)\pm} + \vec{p}\times\vec{g}_{2a}^{(0)\pm} &= (m-\sigma)\vec{g}_{0a}^{(0)\pm}.
\end{aligned}
$$

0th Order Solution

Gap equation:

$$m_0\pi=0,$$

$$(m_0-\sigma)\left(1+G\int\frac{d^3\vec{p}}{(2\pi)^3}\sum_{a=u,d}\frac{f_{0a}^{+}-f_{0a}^{-}}{E_p}\right)-m_0=0$$

Which means pion condensate must be zero in classical limit.
Then we can find the solutions to the equations:

$$f_{1a}^{\pm}=\pm\frac{\vec{p}}{E_p}\cdot\vec{g}_{0a}^{\pm},$$

$$f_{2a}^{\pm}=0,$$

$$f_{3a}^{\pm}=\pm\frac{m_0-\sigma}{E_p}f_{0a}^{\pm},$$

$$\vec{g}_{1a}^{\pm}=\pm\frac{\vec{p}}{E_p}f_{0a}^{\pm},$$

$$\vec{g}_{2a}^{\pm}=\frac{\vec{p}\times\vec{g}_{0a}^{\pm}}{m_0-\sigma},$$

$$\vec{g}_{3a}^{\pm}=\mp\frac{E_p^2(m_0-\sigma)\vec{g}_{0a}^{\pm}-(m_0-\sigma)(\vec{p}\cdot\vec{g}_{0a}^{\pm})\vec{p}}{E_p(m_0-\sigma)^2}.$$

0th Order Solution

Transport equations for f_0 and $\vec{g}_0$:

$$\left(D_a \pm \frac{\vec{p}}{E_p}\cdot\vec{D}_a \mp \frac{\vec{\nabla}m^2\cdot\vec{\nabla}_p}{2E_p}\right) f_{0a}^{\pm} = 0,$$

$$\left(D_a \pm \frac{\vec{p}}{E_p}\cdot\vec{D}_a \mp \frac{\vec{\nabla}m^2\cdot\vec{\nabla}_p}{2E_p}\right) \vec{g}_{0a}^{\pm} = \frac{q_a}{E_p^2}\left[\vec{p}\times\left(\vec{E}\times\vec{g}_{0a}^{\pm}\right) \mp E_p\vec{B}\times\vec{g}_{0a}^{\pm}\right]$$
$$-\frac{1}{2E_p^4}\left(\partial_t m^2\vec{p} \mp E_p\vec{\nabla}m^2\right)\times\left(\vec{p}\times\vec{g}_{0a}^{\pm}\right).$$

Where $m = m_0 - \sigma$.

A homogeneous solution for $\vec{g}_0$:

$$\vec{g}_{0a}^{\pm} = \frac{Q_a}{m^2}\left(\mp\vec{B} + \frac{\vec{p}}{E_p}\times\vec{E}\right).$$

From 1st order equations we can find:

$$\pi^{(1)} = \frac{G}{2m_0}\int\frac{d^3\vec{p}}{(2\pi)^3}\sum_{a=u,d} Q_a\left[(\vec{B}\times\vec{\nabla}_p)\cdot\vec{g}_{0a} + \vec{E}\cdot\vec{\nabla}_p f_{1a}\right]$$
$$= -\frac{G}{4\pi^2 m_0}\left(Q_u^2 - Q_d^2\right)\frac{\Lambda^3}{m^2\sqrt{\Lambda^2+m^2}}\vec{E}\cdot\vec{B}$$

Time Evolution: Demonstration

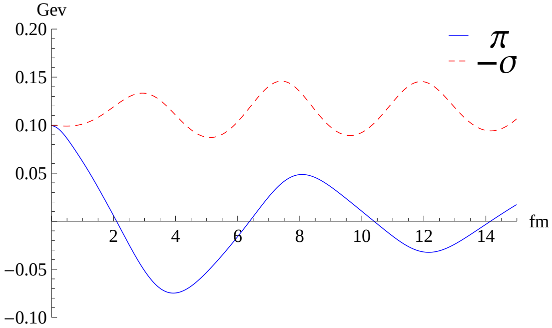


Figure: The time evolution of chiral condensate $\sigma(t)$ (dashed line) and pion condensate $\pi(t)$ (solid line).

1

[1] XG, Pengfei Zhuang, Phys. Rev. D 98, 016007 (2018)

Mass Correction

Assume $\sigma = \pi = 0$ and m to be small. Define right/left hand components

$$\begin{aligned} f_\chi &= f_0 + \chi f_1 \\ \vec{g}_\chi &= \vec{g}_1 + \chi \vec{g}_0 \end{aligned}$$

Massless case:

$$\partial_t f_\chi^\pm + \dot{\vec{x}} \cdot \nabla f_\chi^\pm + \dot{\vec{p}} \cdot \nabla_p f_\chi^\pm = 0$$

$$\dot{\vec{x}} = \frac{1}{\sqrt{G}}(\vec{v}_p + q\hbar(\vec{v}_p \cdot \vec{b})\vec{B} + q\hbar \vec{E} \times \vec{b})$$

$$\dot{\vec{p}} = \frac{1}{\sqrt{G}}(q\vec{v}_p \times \vec{B} + q\vec{E} + q^2\hbar(\vec{E} \cdot \vec{B})\vec{b})$$

Mass Correction

Massive case, keep to the first order of m:

$$\partial_t f_\chi^\pm + \dot{\vec{x}}\cdot\nabla f_\chi^\pm + \dot{\vec{p}}\cdot\nabla_p f_\chi^\pm = -m\chi \frac{q\vec{E}\cdot(\vec{g}_3^{(0)\pm} + \hbar\vec{g}_3^{(1)\pm})}{\sqrt{G}p^2} + m\hbar \frac{F_2\left[\vec{g}_3^{(0)\pm}\right]}{\sqrt{G}}$$

$$F_2\left[\vec{g}_3^{(0)\pm}\right] = \frac{1}{2p^4}(\vec{p}\cdot\vec{\mathrm{D}})(q\vec{B}\cdot\vec{g}_3^{(0)\pm}) \pm \frac{1}{2p^3}\vec{\mathrm{D}}\cdot(q\vec{E}\times\vec{g}_3^{(0)\pm}) \mp \frac{3}{2p^5}(q\vec{B}\times\vec{p})\cdot(\vec{E}\times\vec{g}_3^{(0)\pm})$$

- $\vec{b}$, $\dot{\vec{x}}$, $\dot{\vec{p}}$ not altered
- ‘Dissipation terms’ proportional to m, involving EM field and magnetic momentum $\vec{g}_3$.

Mass Correction to CME

With only B field:

$$\partial_t f_\chi^\pm + \dot{\vec{x}} \cdot \nabla f_\chi^\pm + \dot{\vec{p}} \cdot \nabla_p f_\chi^\pm = m\hbar \frac{1}{2p^4\sqrt{G}} (\vec{p} \cdot \vec{\mathrm{D}})(\vec{B} \cdot \vec{g}_3^{(0)\pm})$$

$$\dot{\vec{x}} = \frac{\vec{p}}{p\sqrt{G}}(1 + 2q\hbar\vec{b} \cdot \vec{B})$$

$$\dot{\vec{p}} = \frac{q}{\sqrt{G}} \vec{v}_p \times \vec{B}$$

Dissipation term

- At the 1st order of $\hbar$.
- Does not dependent on chirality.
- Has formal solution.

Summary

- From the equal-time Wigner function of massless fermions, we derived the transport equation up to first order of $\hbar$,
- The result is in consistent with former results.
- Weryl fermion provides a simpler and clearer method