RADIATIVE BOUND STATE FORMATION IN SUPERSYMMETRY

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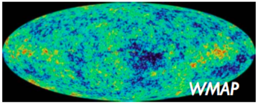
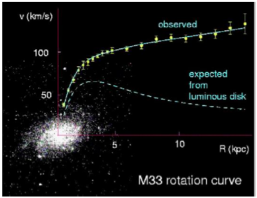
Cold Collisionless Dark Matter Paradigm

- 27% of the Universe is dark matter (DM)
- All evidence for DM comes through its gravitational effect in the Universe
- Large scale structure of the Universe well described by cold, collisionless DM – i.e. typical WIMP-Neutralino



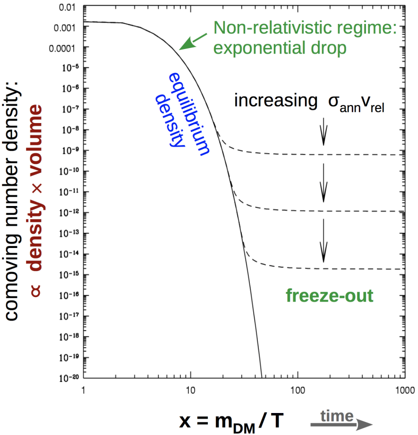


Relic density of DM with contact interactions

- Early unvierse: Hot theraml bath of elementary particles: $n_\chi = n_\chi(T)$ kept in chemical equilibrium via annihilation, $\chi + \bar{\chi} \to f + \bar{f}$
- As universe expands and cools, annihilations become inefficient. Exponential decrease of $n_\chi(T) \to$ freeze-out yields relic density

$$\Omega_\chi = 0.26 \times \left(\frac{3 \times 10^{-26} cm^3/s}{\langle \sigma v \rangle} \right)$$

velocity independent cross section



Small Scale Anomalies: Self-interacting DM

failure of CCDM-only simulations

- Self-interacting DM can solve all problems
  - Core vs cusp: energy transfer from outer to inner halo
  - Missing satellites: smaller halos stripped in hot MW halo
  - Too big to fail: rotation curves modified by self-interactions
- The required scattering cross section is huge

$$\frac{\sigma}{m_\chi} \sim 1\text{cm}^2/g$$

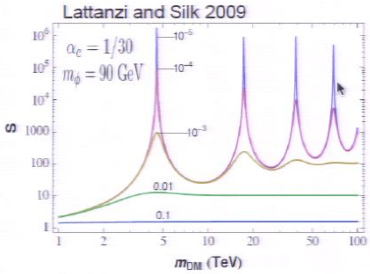
Short-range DM Tension-I

Self-interacting DM suggests a ultra-light force mediator to boost dark matter self-scattering $\rightarrow$ Long-range interaction

Cosmic Ray Anomalies: Sommerfeld Effect

failure of DM with velocity independent cross section

- WIMP such as neutralino annihilation can explain cosmic ray anomalies but large cross section 100 times over thermal relic value.
- A new force mediator acting between WIMPs enhances the cross section via sommerfeld enhancement



Short-range DM Tension-2

Sommerfeld enhancement requires light mediator $\rightarrow$ Long-range interaction

Last Reason

- Most DM research has focused on contact interactions. Prototypical WIMP scenario: $m_{\mathsf{DM}} \sim m_{\mathsf{mediator}} \sim 100$GeV. Why not long-range interaction?
- Long range interaction implication: Sommerfeld enhancement and Bound state
- Bound state implication: New radiative processes and modified rates.

Toy Model

General NMSSM for solving SIDM

NMSSM Interactions

Superpotential

Superpotential determines the supersymmetric interaction between matter superfields

$$W = Y_u^{ij}\hat{U}_i\hat{Q}_j\hat{H}_u - Y_d^{ij}\hat{D}_i\hat{Q}_j\hat{H}_d - Y_e^{ij}\hat{E}_i\hat{L}_j\hat{H}_d + \mu\hat{H}_u\hat{H}_d$$
$$\frac{1}{2}\mu_s S^2 + \frac{1}{3}\kappa S^3 \,.$$

Soft Terms

Soft terms provide non-supersymmetric interaction between scalars and gauginos

$$\mathcal{L}_{NMSSM} = \mathcal{L}_{MSSM} + \frac{1}{3}A_\kappa S^3 + \cdots$$

Long-range interaction in NMSSM

Mediator Classification: CP-even and CP-odd singlet

$S = \phi + i\sigma$

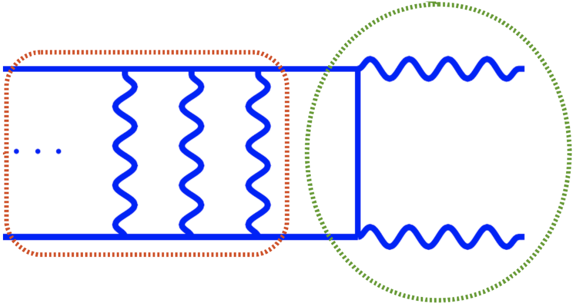
Why only consider CP-even mediator ϕ

- CP-odd mediator interaction is spin dependent.
- Constrained by CMB

Only CP-even ϕ is considered

$V = -\frac{\alpha e^{-m_\phi r}}{r}$ and $\alpha = \frac{\kappa^2}{4\pi}$

P-wave sommerfeld enhancement



$$\mathcal{M} = \int \frac{d^3p}{(2\pi)^3} \Phi(p) A_4$$

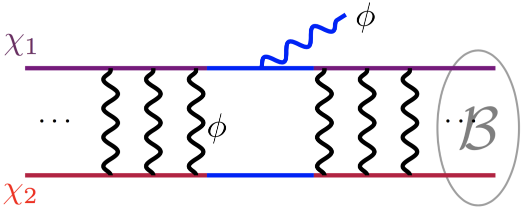
P-wave seems safe under CMB constraint

What about bound state

Bound State Definition

Process: $\mathcal{U}\{\chi_1 + \chi_2\} \rightarrow \mathcal{B}\{\chi_1\chi_2\} + \phi$

$\mathcal{U}$ is scattering unbounded states, $\mathcal{B}$ is bound state, ϕ is light CP-even singlet.



Difficulty

double re-summation compared with sommerfeld enhancement

A Basic Prescription of Radiative capture in QFT

four "easy" steps

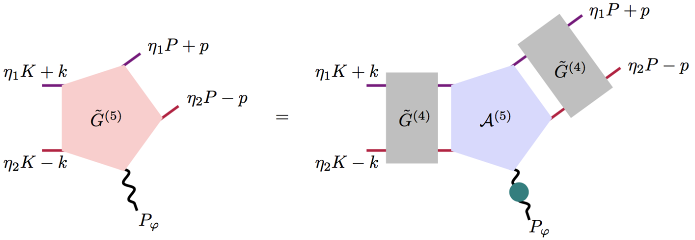
- Separate the asymptotic states from the interaction part (hard scattering process) factorization?
- Compute the properties of the asymptotic states i.e. Bethe-Salpeter equation.
  - Scattering state $\mathcal{U}$ is initial state
  - Bound state $\mathcal{B}$ is final state: one particle state
- Extract the amplitude from LSZ reduction formula.
  - G_5 is related with S-matrix via LSZ reduction formula
  - G_5 is a product of G_4 and A_5
  - Equivalence gives rise to relationship between S-matrix and A_5
- Compute the interaction part in terms of Feynman diagrams.

Identify and separate the asymptotic states

Step-1

5-point correlation function

$$G_5 = \langle \Omega | T\phi(X)\chi_1(x_1)\chi_2(x_2)\bar{\chi}_1(y_1)\bar{\chi}_2(y_2)|\Omega\rangle$$



Decompose the five-point function into a fully amputated hard scattering part A_5 and four-point correlation function G_4.

4-point correlation function vs Bethe-Sapeter wavefunction

Schematic form of the 4-point correlation function

$$G_4 = \langle \Omega | T\chi_1(x_1)\chi_2(x_2)\bar{\chi}_1(y_1)\bar{\chi}_2(y_2)|\Omega\rangle$$

- Insert unity operator in the 4-point function:

$$1 \sim \sum_n \int d^3Q|\mathcal{B}\rangle\langle\mathcal{B}| + \int d^3q d^3Q|\mathcal{U}\rangle\langle\mathcal{U}|$$

- Pick out a state in which singularity dominates $\rightarrow$ Poles

$$G_4 \sim \sum_n \frac{i\Psi_{Q,n}(x)\Psi^\star_{Q,n}(y)}{Q^0 - \omega_{Q,n} + i\epsilon} + \int d^3q \frac{i\Phi_{Q,q}(x)\Phi^\star_{Q,q}(y)}{Q^0 - \omega_{Q,q} + i\epsilon}$$

Bethe-Salpeter wavefunctions

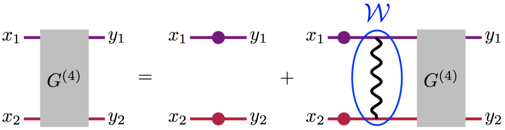
$$\Psi_{Q,n}(x_1,x_2) = \langle\Omega|T\chi_1(x_1)\chi_2(x_2)|\mathcal{B}\rangle,\ \Phi_{Q,q}(x_1,x_2) = \langle\Omega|T\chi_1(x_1)\chi_2(x_2)|\mathcal{U}\rangle.$$

Solving asymptotic states

Step-2

Recursion relation

4-point function can be solved in Dyson-Schwinger equation



Replacing G_4 by Bethe-Salpeter wavefunctions yields Bethe-Salpeter equations.

$$\tilde{\Psi}_{Q,n}(p) = S(p,Q)\int \frac{d^4k}{(2\pi)^4}\tilde{W}(p,k,Q)\tilde{\Psi}_{Q,n}(k)$$

$$\tilde{\Phi}_{Q,q}(p) = S(p,Q)\int \frac{d^4k}{(2\pi)^4}\tilde{W}(p,k,Q)\tilde{\Phi}_{Q,q}(k)$$

Reduce Bethe-Salpeter Equation into Schrodinger Equation

Instaneous approximation

$\tilde{W}(p,k,Q) \sim \tilde{W}(p,k)$

together with non-relativistic limit give rise to Schrodinger equation

$$\left(-\frac{\nabla^2}{2\mu} + V(r)\right)\psi_n(r) = -\mathcal{E}_n\psi_n(r),$$

$$\left(-\frac{\nabla^2}{2\mu} + V(r)\right)\phi_q(r) = \mathcal{E}_q\phi_q(r).$$

with potential generated by

$$V(r) = -\frac{1}{i4\mu m}\int \frac{d^3k}{(2\pi)^3}\tilde{W}(k)\exp(ikr)$$

LSZ reduction: From Green function to S-matrix

Step-3

- In terms of LSZ reduction formula, Fourier transformation of five point Green function is equivalent to S-matrix.
- G_5 is also a product of G_4 and A_5.
- Combining them yields a simple relation for the scattering matrix element

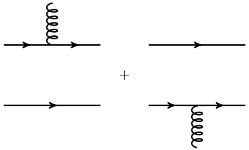
$$\mathcal{M}_{k\to\{nlm\}} = \int \frac{d^4p}{(2\pi)^4} \Psi_n(p)\Phi_k(q)A_5$$

A_5 is the perturbative transition process

It can be computed via Feynman diagram!

Computing the hard process A_5

Step-4



- matrix element

$$\mathcal{M}_{k\to nlm} = M\sqrt{\mu}\int d^3r\psi^\star_{nlm}(r)\phi_k(r)(g_1e^{(-i\eta_2P_\phi r)} + g_2e^{(-i\eta_1P_\phi r)})$$

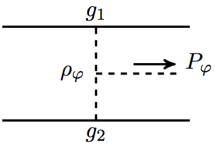
- The cross section for transition into ground state $\{nlm\} = \{100\}$

$$\sigma_{\text{BSF}}v_{\text{rel}} = \left[\frac{(g_1\eta_2 - g_2\eta_1)^2}{16\pi\alpha}\right]\frac{\pi\alpha^2}{\mu^2}S_{\text{BSF1}}$$

Accidental cancellation for singlino DM

$g_1 = g_2 = \kappa, \quad \eta_1 = \eta_2 = 1/2$

Way out: self-interaction of singlet A_κ



- For majorana singlino, the bound state formation cross section is

$$\sigma_{\mathrm{BSF}} v_{\mathrm{rel}} = \frac{1}{2}\frac{\alpha^3}{4\mu^2}\left(\frac{A_\kappa}{\mu\alpha^2/2}\right)^2 S_0[\zeta,\xi]\left(\frac{\zeta^2}{1+\zeta^2}\right)^3 e^{-4\zeta \mathrm{arccot}\zeta}$$

Implication on DM relic density

provide a new channel for annihilation

Enlarged Boltzmann equations

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -(n_\chi^2 - n_\chi^{eq2})\langle\sigma_{\mathsf{ann}} v_{\mathsf{rel}}\rangle - n_\chi^2\langle\sigma_{\mathsf{BSF}} v_{\mathsf{rel}}\rangle + n_{\mathcal{B}}\Gamma_{\mathsf{ion}}$$
$$\frac{dn_{\mathcal{B}}}{dt} + 3Hn_{\mathcal{B}} = n_\chi^2\langle\sigma_{\mathsf{BSF}} v_{\mathsf{rel}}\rangle - n_{\mathcal{B}}(\Gamma_{\mathsf{ion}} + \Gamma_{\mathsf{decay}})$$

Precision Calculation Era for DM?

What about loop correction, multiple mediator emission, direct detection